REVIEW FOR EXAM 1

1. A home buyer takes out a loan at 6% interest rate. She will repay the loan in 20 years with a monthly payment of $800. Assume that the interest is compounded continuously and that payments are made continuously.

   (a) Let \( l(t) \) be the amount of loan owed at time \( t \) (in years). Write down the differential equation and initial condition for \( l(t) \).

   (b) Find \( l(t) \).

   (c) What is the total amount of the loan?

   (d) What is the total interest payed during 20 years?

2. A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors the population will triple in two weeks time. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes. If there are initially 100 insects in the area will the population survive? If not, when do they die out?

3. (a) Solve the differential equation \( 2 \frac{dy}{dt} = 3t - y, \ y(0) = y_0 \).

   (b) Determine how different values \( y_0 \) affect \( \lim_{t \to \infty} y \).

4. (a) Solve the differential equation \( t \frac{dy}{dt} = \sin(t) - 2y, \ y(\frac{\pi}{2}) = y_0 \ (t > 0) \).

   (b) Determine how different values \( y_0 \) affect \( \lim_{t \to \infty} y \).

5. Solve the differential equation \( \frac{dy}{dx} = \frac{x^2}{y(1 + x^3)} \) explicitly for \( y \).

6. Solve the differential equation \( \frac{dy}{dx} = 2y^2 + xy^2 \) explicitly for \( y \).

7. Solve the differential equation \( 6 \frac{dy}{dx} = xy^2 + 2y \) explicitly for \( y \).

8. Solve the differential equation \( \frac{dy}{dx} = \frac{x^2 - 3y^2}{2xy} \) explicitly for \( y \).
9. Match the differential equation with the corresponding direction field. **Show all work!**

(a) \( \frac{dy}{dt} = y - 2 \)

(b) \( \frac{dy}{dt} = 2 - y \)

(c) \( \frac{dy}{dt} = y(y - 2) \)

(d) \( \frac{dy}{dt} = y(2 - y) \)
Extra Credit:

10. Solve the differential equation $x \frac{dy}{dx} = y(\ln(x) - \ln(y))$. 