1. Evaluate the following limits (if they exist):
   a) \( \lim_{x \to 5} \frac{x^2 - 4x - 5}{x - 5} \)
   b) \( \lim_{x \to 0} \frac{\sin x}{3x} \)
   c) \( \lim_{x \to 1} \frac{|x - 1|}{x - 1} \)

2. Use the definition of derivative to find the derivative of \( f(x) = x^2 - 3x - 7 \).

3. Find the derivatives of the following functions:
   a) \( f(x) = \frac{2x}{3x^2 - 4} \)
   b) \( y = \sqrt[3]{6x^2 - 4} \)
   c) \( y = \frac{4}{(x^2 - 1)^2} \)
   d) \( y = x^7 \cos \pi x \)

4. Find an equation of the line tangent to \( y = x^3 - 3x^2 + 4 \) at the point (1, 2).

5. At what point(s) does \( y = x^3 - 3x \) have a horizontal tangent?

6. Find the slope of the line tangent to \( x^3 - 2x^2 + y^2 = 17 \) at (2, -1).

7. The radius \( r \) of a circle is decreasing at the rate of 2 inches per minute. Find the rate of change of the area of the circle when \( r = 3 \) inches.

8. Verify Rolle's Theorem for \( f(x) = x^2 - 6x + 5 \) for the interval [1, 5].

9. Apply the Mean Value Theorem to \( f(x) = x^2 - 4 \) for the interval [1, 4].

10. For the following functions, find all relative extrema, points of inflection and sketch the graph of the function:
    a) \( g(x) = x^3 - 3x^2 - 9x + 1 \)
    b) \( f(x) = 4 \sin x \) \( [0 \leq x < 2\pi] \)

11. Sketch the following graphs. In each problem, find any intercepts, asymptotes, relative extrema and points of inflection.
    a) \( y = \frac{1}{4 - x^2} \)
    b) \( y = \frac{4}{x^2 + 4} \)
    c) \( y = \frac{x}{x^2 - 1} \)
    d) \( y = \frac{x + 1}{(x - 1)^2} \)

12. A farmer wants to fence in 48,000 square feet of land in a rectangular plot and then divide it into three parts (as pictured below) by creating a fence at the middle of the plot parallel to one of the sides and then subdividing that part in 1/2. What is the minimum amount of fencing needed to accomplish this task?

13. Integrate:
   a) \( \int (x^2 - 5x - 3) \, dx \)
   b) \( \int \frac{x^2 - 1}{x^2} \, dx \)
   c) \( \int (\cos x - \csc^2 x) \, dx \)
   d) \( \int \frac{x}{\sqrt{x^2 - 1}} \, dx \)
   e) \( \int \sin x \cos x \, dx \)
   f) \( \int x \sqrt{x + 1} \, dx \)

14. Evaluate:
   a) \( \int_{0}^{5} \sqrt{3x + 1} \, dx \)
   b) \( \int_{1}^{3} (3x^2 - 2x + 7) \, dx \)
   c) \( \int_{0}^{\pi/6} \sin 2x \, dx \)
15. Sketch the region bounded by the graphs of the functions and find the area of the region: \( y = x^2 - x - 6 \) and \( y = x - 3 \).

16. Find the area of the region bounded by \( y = x^3 - x \) and \( y = 0 \).

17. Find the area of the region bounded by \( y = x^2 + 2x \), \( y = 0 \), \( x = -2 \).

18. Find the area of the region bounded by \( x = y^2 - 2y \) and \( x = 0 \).

19. Find the area of the region bounded by \( x = y^2 - 3y \), \( x = 0 \), \( y = -1 \), \( y = 2 \).

20. A spherical balloon is inflated with helium. The rate of change of the radius is 3 ft/min. Find the rate of change of the volume where
   
   (a) \( R = 2 \) ft          
   (b) \( V = 36\pi ft^3 \)

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**MA475 Final Review Answers.**

1. (a) 6          
   (b) \( \frac{1}{3} \)          
   (c) doesn’t exist

2. \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = -2x - 3 \)

3. (a) \( f'(x) = \frac{-6b - 8}{(3x^2 - 4)^2} \)          
   (b) \( y' = \frac{4x}{\sqrt[3]{(6x^2 - 4)^2}} \)

   (c) \( y' = \frac{-16x}{(x^2 - 1)^3} \)          
   (d) \( y' = 2x \cos \pi x - \pi x^2 \sin \pi x \)

4. \( y = -3x + 5 \)

5. (1, -2) and (-1, 2)

6. \( y' = \frac{4xy - 3x^2}{2y - 2x^2} \) at (2, -1), \( y' = 2 \)

7. \( -12\pi \) in\(^2\)/min

8. \( f(1) = 0 \), \( f(5) = 0 \), \( f'(c) = 0 \), \( f'(x) = 2x - 6 \), \( x = 3 \), \( c = 3 \)

9. \( f(x) = x^2 - 4 \), \( f'(x) = -3 \), \( f(4) = 12 \), \( f'(c) = \frac{12 - (-3)}{4 - 1} = 5 \), \( f'(x) = 2x - 5 \), \( x = 2 \frac{1}{2} \), \( c = 2 \frac{1}{2} \).
10. (a) max (-1, 6), min (3, -26), Inflection point (1, -10)

(b) max ($\frac{\pi}{2}$, 4), min (3$\frac{\pi}{2}$, -4), Inflection points (0,0), ($\pi$, 0)

11. a. x-intercept: none, y-intercept: ¼, horizontal asymptote: y=0, vertical asymptote: $x = \pm 2$, max (0,1/4), min: none, points of inflection: none.
b. x-intercept: none, y-intercept: 1, horizontal asymptote: \( y=0 \), vertical asymptote: none, max \((0,1)\), min: none, points of inflection: \((-\frac{2\sqrt{3}}{3}, \frac{3}{4})\) and \((\frac{2\sqrt{3}}{3}, \frac{3}{4})\).

c. x- and y–intercept: 0, horizontal asymptote: \( y=0 \), vertical asymptote: \( x = \pm 1 \), no max and min, point of inflection: \((0,0)\).
d. x-intercept: -1, y-intercept: 1, horizontal asymptote: y=0, vertical asymptote: \( x = 1 \), no max, local min \( \left( -3, -\frac{1}{8} \right) \) and point of inflection \( \left( -5, -\frac{1}{9} \right) \).

12. length=240 ft, width=200 ft, minimum amount=1200 ft.

13. (a) \( \frac{x^3}{3} - \frac{5x^2}{2} - 3x + c \)  
(b) \( x + \frac{1}{x} + c \)  
(c) \( \sin x + \cot x + c \)  
(d) \( \sqrt{x^2 - 1} + c \)  
(e) \( \frac{\sin^2 x}{2} + c \)  
(f) \( \frac{2(x+1)^{3/2}}{15} [3x-2] + c \)
14. (a) \( \frac{126}{9} \)  
    (b) 32  
    (c) \( \frac{1}{4} \)

15. \( \frac{2}{3} \)

16. \( \frac{1}{2} \)

17. \( 2 \frac{2}{3} \)

18. \( \frac{4}{3} \)

19. \( \frac{31}{6} \)

20. (a) \( 48 \pi \text{ ft}^3 / \text{min} \)  
    (b) \( 108 \pi \text{ ft}^3 / \text{min} \)