The Life and Work of Fiona Murnaghan

By Adelle Thomas

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Introduction

Dr. Fiona Murnaghan is a Professor of Mathematics at the University of Toronto. Since earning her PhD from the University of Chicago in 1987, she has made many important contributions to the field of Representation theory of p-adic groups. We examined her rise to success from grade school, through graduate school and finally as a Professor of Mathematics. The first section of the poster includes our biography of Dr. Murnaghan. The next section is a selection of results we learned in the general area in which Dr. Murnaghan’s research lies, group theory. Her actual field of study within group theory was well beyond the scope of this project, instead we examined a classical result in group theory: A description of the symmetry groups of platonic solids.
Section 1: Biography of Fiona Murnaghan

By Adelle Thomas and Andrew Douglas

Fiona Murnaghan was a bright, curious and creative child who always loved school, and especially mathematics. Although growing up she never planned to be a mathematician, the subject offered intellectual and creative challenges that no other field could match. Now a full professor of mathematics at the University of Toronto, she is a well respected expert in the field of Representations of reductive p-adic groups, and a talented teacher.

Murnaghan was born in London, England in 1960 to an Irish father, and a Scottish mother. During her early years her family moved frequently. When she was a baby her parents moved to Belfast and then Armagh in Northern Ireland. When she was almost 9 years old, her parents moved to Canada. They lived in Vancouver, Edmonton and then finally settled on Vancouver Island in a small town named Campbell River.

Murnaghan grew up in an intellectually stimulating environment. Her father was an architect. He was independent and preferred to work for himself which lead him to start his own architectural company. Her mother was a sculptor, and also worked as an elementary teacher for 5 years, and an activity planner for a hospital for 20 years. They encouraged her to follow her interests, be they artistic or scientific, although her mother disliked mathematics. It was, perhaps, this exposure to both art and science, and the influence of her father’s independent spirit that lead her to a career in mathematics. After all, it is the creative, yet logical and independent mind that is so ubiquitous in mathematics.
Murnaghan did her undergraduate degree at the University of British Columbia, in Vancouver. She began her Bachelor of Science degree in 1978, and immediately found an affinity for mathematics. She took as many math courses as was allowed, saying that “math was by far the most interesting of all the subjects.” In the final year of her B.Sc. degree, she still did not know which profession to pursue. Her mother was worried that she would have trouble finding a career in mathematics. But, ultimately, her parents recognized her love of mathematics and supported and encouraged her to study math in graduate school.

In 1982 Murnaghan entered the graduate mathematics program at the prestigious University of Chicago. She became immersed in the academic world; it was here that she realized that mathematics was the career that she was meant to follow. Guided by the mathematician Paul Sally, Jr, she finished her doctoral thesis entitled “Invariant Meromorphic Distributions of p-adic GL(n)” in 1987. The thesis resulted in her first publication with the same title in the American Journal of Mathematics in 1989. Shortly after graduating, she earned a position as an Assistant Professor at the University of Toronto, ending her mother’s worries about her finding a job in academia.

Murnaghan has more than 25 research articles; an impressive number in a field of expertise not known for prolific publication due to its difficulty, and depth of knowledge that is required to tackle any problem in the area. Her articles have been published in such journals as the American Journal of Mathematics (mentioned above), the Pacific Journal of Mathematics, and Mathematics Annals. Although she is most often the sole author of her articles, especially at the beginning of her career, she has published in collaboration with such
mathematical heavy weights as Jim Arthur (former president of the American Mathematical Society). She describes the collaborative process in mathematics as an aspect that greatly attracts her to the profession.

Her area of specialty is Representations of reductive p-adic groups. Although it is beyond the scope of this biography to give a complete explanation of the specialty, a partial description is provided. Groups are abstract algebraic structures. Group theory can be considered as the study of symmetry: the collection of symmetries of some object preserving some of its structure forms a group. Reductive p-adic groups are very complicated types of groups. Finally, representation theory is a branch of mathematics that studies properties of abstract groups via their representation as linear transformations of vector spaces. That is, representation theory reduces a problem in group theory, which can be extremely difficult, to a problem of linear algebra, a field in which much is known and which is much easier to handle.

In addition to her love of research, she greatly enjoys passing on her love of mathematics to her students. Her student reviews, as published online, tend to be exceptional, with the word ‘amazing’ appearing often. Andrew Douglas, now an Assistant Professor at CUNY, was former PhD student that she helped to supervise. He describes his experience working partially under her supervision as “a very positive experience. She clearly genuinely cares about her students. Also, she has very brilliant insights into any problems that you may have.”

Somewhat to our surprise, Murnaghan seems to have faced few obstacles succeeding in mathematics resulting from her being a woman. However, she does sight antiquated attitudes about woman still existing in some parts of society that
work to discourage woman from entering mathematics. As Murnaghan puts it, “traditionally women were not viewed as fully human, with their own imagination and mental capacities. I expect this has a lot to do with (negative attitudes towards women in math).” She also notes the isolation that woman can sometimes feel in math department which have traditionally been male dominated. However, she points out that men do not numerically dominate math departments as they once did, and the numerical gap is decreasing practically every year, at least at the University of Toronto. So, isolation from other female mathematicians is not a serious issue in her career.

Professor Fiona Murnaghan is a highly successful mathematician, and a gifted teacher. Her success in mathematics, and the enjoyment she derives from the subject, teaching and mentoring students is sure to inspire generations of students to pursue careers in mathematics, and math related fields.

References

Section 2: Group Theory

Introduction:

Group theory is a powerful formal method for analyzing abstract and physical systems in which symmetry is present. As a result, group theory has become ubiquitous in many areas of physics (most notably quantum mechanics), chemistry, and biology.

Definition: A group \((G, *)\) is a set \(G\) closed under a binary operation \(*\) satisfying the following 3 axioms:

- **Associativity:** For all \(a, b, c \in G\), \((a * b) * c = a * (b * c)\).
- **Identity element:** There exists an \(e \in G\) such that for all \(a \in G\), \(e * a = a * e = a\).
- **Inverse element:** For each \(a \in G\), there is an element \(b \in G\) such that \(a * b = b * a = e\), where \(e\) is an identity element.

Examples:

1. **The Symmetry Group of an Equilateral Triangle:** The set of motions that can bring an equilateral triangle back to its original position form a group. There are six such motions as listed below:

   - Do nothing
   - Rotate 120 degrees counterclockwise
   - Rotate 240 degrees counterclockwise
   - Flip about the symmetry axis through the upper vertex
   - Flip about the symmetry axis through the lower left-hand vertex
   - Flip about the symmetry axis through the lower right-hand vertex

The 6 symmetry motions are pictured in order below:
We use the operation of followed by and find that the axioms for a group hold:

- **Closure**: performing one motion followed by performing another motion is equivalent (has the same effect) as performing one of the 6 motions.
- **Associativity**: since followed by is always an associative operation.
- **Identity**: The Do nothing motion is the identity element.
- **Inverses**: Each element has an inverse:
  - The Do nothing is its own inverse.
  - The **Rotate 120 degrees** and the **Rotate 240 degrees** are inverses of each other.
  - The three **Flip** movements are their own inverses.

Each of the six symmetries can be seen as a permutation of the set \{a,b,c\}. From here it can be shown that the symmetry group of an equilateral triangle is the permutation group S3.
2. The group of rotations of a square:

The group is generated by the rotation \( r = 90 \) degrees. There are three other elements in this group: \( r^2 = 180 \) degree, \( r^3 = 270 \) degrees, \( r^4 = 360 \) degrees \( \sim 0 \) degrees. This is the cyclic group of order 4, denoted \( Z_4 \).

\[ \begin{array}{cccc}
A & B & D & A \\
D & C & C & B \\
B & A & D & B \\
A & D & B & A \\
\end{array} \]

3. Symmetry of \( \text{Cr}_7\text{NiF}_8(\text{O}_2\text{CCMe}_3)_{16} \)
The symmetry of a molecule reveals information about its properties (i.e., structure, spectra, polarity, chirality, etc.). The above molecule has the same symmetry group as the octagon (we disregarded color). The octagon has 16 different symmetries: 8 rotational symmetries and 8 reflection symmetries. The associated rotations and reflections make up the dihedral group $D_8$. The following picture shows the effect of the sixteen elements of $D_8$ on a stop sign:
Section 3: Symmetry Groups of the Platonic Solids

Introduction

The inhabitants of Neolithic Scotland, some 3000 years ago, were the first to develop the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron (Atiyah and Sutcliffe 2003). Their stone models of these solids are kept in the Ashmolean Museum in Oxford, England. Two thousand years later, in ancient Greece, these solids again appear. This time in Plato’s Timaeus ca. 350 BC (Cromwell 1997). From then on, they have been called the Platonic Solids.

Platonic Solids from Neolithic Scotland

Mathematically, a platonic solid is a 3-dimensional convex shape (no "dents" or indentations) with each face being a regular polygon of the same size and shape. Euclid proved in the last proposition of Elements that only the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron satisfy the definition of a platonic solid.

The platonic solids exhibit remarkable symmetry. Below, we describe the symmetry group of each platonic solid. We show the derivation in the easiest case, the tetrahedron.
### Legend:
- $S_4$ is the permutation group on four elements.
- $Z_2$ is the cyclic group of order 2.
- $A_5$ is the alternating group.

<table>
<thead>
<tr>
<th>Platonic Solid</th>
<th>Symmetry Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrahedron</td>
<td>$S_4$</td>
</tr>
<tr>
<td>Cube and Octahedron</td>
<td>$S_4 \times Z_2$</td>
</tr>
<tr>
<td>Icosahedron and Dodecahedron</td>
<td>$A_5 \times Z_2$</td>
</tr>
</tbody>
</table>

### The Symmetry Group of the Tetrahedron

There are two types of symmetry: rotations and reflections.

**Rotations**

There are 12 rotational symmetries. In Figure 1, consider the rotational axis $OA$, which runs from the topmost vertex through the center of the base. Note that we can repeatedly rotate the tetrahedron about $OA$ by $360 \div 3 = 120$ degrees to find two new symmetries. Since each face of the tetrahedron can be pierced with an axis in this fashion, we have found $4 \times 2 = 8$ new symmetries.
Now consider rotational axis $OB$, which runs from the midpoint of one edge to the midpoint of the opposite edge. Rotating 180 degrees about this axis produces another symmetry. In figure 1, the function $p$ describes the tetrahedron's new orientation. Since there are three pairs of opposing edges whose midpoints can be pierced in this fashion, we have found 3 additional symmetries. Finally, we count the identity relation (no rotation) as a symmetry, yielding a total of 12 rotational symmetries along seven different axes of rotation.

**Reflections**

We can find one new symmetry by reflection in the shaded plane in Figure 2. Denote this odd permutation by $s$, and note that $s^2 = e$ (the identity). By composing each of the rotational symmetries with $s$, we can now find 12 new symmetries.

**Conclusion**

We have found that there are 24 symmetries. Each symmetry may be thought of as a permutation of the vertices as labeled by the set \{1, 2, 3, 4\}, and hence an element of $S_4$. $S_4$ has $4! = 24$ elements, and hence each element of $S_4$ has been “found.” Thus the symmetry group of the tetrahedron is $S_4$. 

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**Figure 1.**

**Figure 2.**

$p(1)=4$
$p(2)=3$
$p(3)=2$
$p(4)=1$


References


Additional Figures from Poster

Figure 1: Platonic Life

Figure 2: Hecatonicosachoron
Figure 3: Iron Pyrite form crystals shaped like dodecahedra