4. DYNAMICS: NEWTON’S LAWS OF MOTION.

Key words: Dynamics, Force, First Newton’s Law of Motion, Inertial Reference Frames, Mass, kilogram, Newton’s Second Law of Motion, Newton’s Third Law of motion, Key words, Force of Gravity, Weight, Normal force, Tension Force, Friction Force, Kinetic Friction, Coefficient of Kinetic Friction, Static Friction, Coefficient of Static Friction, Free—body Diagram or Force Diagram.

We are ready to make next step now: to transfer from Kinematics to Dynamics. Dynamics is the branch of Mechanics that relates the geometrical characteristics of motion with the causes of motion. The causes of the motion are forces. The Force in Mechanics is considered as any kind of a push or a pull on an object. It is a vector quantity. We designate it by the capital letter \( F \). Several forces may simultaneously act on the same object. Using the vector addition, we can find total or net force.

\[
\text{F}_{\text{net}} = \Sigma F = F_1 + F_2 + F_3 + \ldots \ldots \ldots \ldots (4.1)
\]

Where Greek letter \( \Sigma \) means sum.

4.1 Newton’s First Law of Motion.

The Newton’s Laws are the core of dynamics. The First Law is about situation when no net force acting on an object. From ancient times, it was supposed that object eventually comes to rest, so the rest is the natural state of each object. Galileo revised this concept. He introduced the concept of the frictional force and stated that when there is no this force, the motion with the constant velocity will be also natural state for each object. This concept incorporated into

**First Newton’s Law of Motion:** A body acted on by no net force moves with constant velocity (which may be zero).

Constant velocity means zero acceleration. Finally the 1\textsuperscript{st} Newton’s Law can be analytically written as following:

\[
\text{If} \Sigma F = 0, \text{then} \ v = \text{const,} \ a = 0\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.2)
\]

It is convenient to write first equation in components:
\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \quad \text{(4.2a)} \]

The tendency of a body to keep moving once it is in motion (or to be at rest if it was initially at rest) results from a property that is called **inertia**. Because of this the 1\textsuperscript{st} Newton’s Law of motion sometimes is called the Law of Inertia. The reference frames in which the law of inertia is valid are called an **Inertial Reference Frames**.

Along with a force, a very important in dynamics is the physical quantity called **Mass**. Newton treated mass as quantity of matter. Mass actually can be considered as the measure of the inertia of an object. The greater mass an object has, the greater force is needed to set this object in motion or to stop it. For example, a huge truck much more difficult to stop than a small compact car. **Mass is scalar, the unit to measure mass in the SI system is kilogram**.

**4.2 Newton’s Second Law of Motion.**

We can perform the simple experiments applying to object different forces. We will see that greater the force, greater the acceleration of an object. If we apply the same force to the objects with different masses, we will deduce that greater the mass of an object, smaller the acceleration. Based on the experiment observation The **Newton’s Second Law of Motion** can be formulated as following.

**The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object.**

The 2\textsuperscript{nd} Law can be written as following:

\[ a = \Sigma \frac{F}{m} \quad \text{(4.3)} \]

or in more wide used form:

\[ \Sigma F = ma \quad \text{(4.4)} \]

From the left side of the Eq. (4.4) we have a vector addition. To find it we use the method of components (see Chapter 3); therefore it is convenient to write Eq. (4.4) in components. For 3D case we will have:

\[ \Sigma F_x = \max \quad \Sigma F_y = \may \quad \Sigma F_z = \maz \quad \text{(4.5)} \]
Using dimensional analysis we can determine from Eq. (4.4) the SI system unit for force and introduce physical quantity **Force** to full extent. It vector quantity. SI unit of force is called after Isaac Newton **newton**:

\[ 1 \text{ newton} = 1 \text{ N} = 1 \left( \frac{\text{kg m}}{\text{s}^2} \right). \]

4.3. Newton’s Third Law of Motion.
Forces are causes of motion, but where do forces come from? They come from the interaction between objects. Forces always come in pairs. Force acting on an object is always the result of its interaction with another object. Newton supposed that two interacting objects must be treated on an equal basis and formulates his

**Newton’s Third Law of Motion** as following.
The forces with two objects interact with each other are equal in magnitudes and opposite in directions. These two forces act on different objects.

If object 1 exerts a force on object 2 (an “action” \( \mathbf{F}_{1\rightarrow 2} \)), then object 2 exerts a force on object 1 (a “reaction” \( \mathbf{F}_{2\rightarrow 1} \)). In this case the 3\(^{rd}\) Law can be written as

\[ \mathbf{F}_{1\rightarrow 2} = -\mathbf{F}_{2\rightarrow 1} \quad (4.6) \]

Sometimes this law is interpreted as “to every action there is an equal and opposite reaction”. But we should remember that two forces described in this law act on different objects.

4.4. Forces in Mechanics.

Analysis of the Newton’s Laws of motion show, that if we know forces acting on an object and its mass (dynamical physical quantities), we, in principle, can solve corresponding equations and find acceleration of the object (kinematical characteristic). Therefore Newton’s Laws of motion relates causes of motion and resulting motion. But, before we try to use this approach, we need to specify forces that usually can be applied to the objects in mechanics.

**Force of Gravity.**
When object is in free fall (undergoes force of gravity) near the surface of the Earth, it is moving with the constant acceleration – acceleration due to gravity \( g \). Using the 2\textsuperscript{nd} Newton’s Law of Motion, we can write for the magnitude of the gravitational force following expression:

\[
F_G = mg
\]  

(4.7)

**Direction of the force of gravity** is always down. The force of gravity often is called **Weight**. In every day life the concepts of weight and mass are often to get confused. For example, there is serious problem in many countries, which is called “how to lose weight”. In physics, solution of the problem “how to lose weight” very simple. Take a look on Eq. (4.7); person can lose the weight by coming to the places where \( g \) is smaller. In New York City \( g = 9.803 \text{ m/s}^2 \), but in the vicinity of the equator \( g = 9.780 \text{ m/s}^2 \), on the Moon surface \( g \) six times smaller than on the Earth surface. Coming to the places with smaller \( g \), person really will lose weight. But there will be no satisfaction. Therefore the right title of this complicated problem should be “how to lose mass”. You should remember, that weight is the force of gravity, vector quantity directed in space and measured in newtons. Mass, from other hand, is scalar, amount of matter in an object measured in kilograms. Considering forces acting on an object, remember that if object has mass, there is the gravitational force acting on it at the earth surface.

**Normal Force.**

When can observe a lot of objects at rest that are situated on some surfaces. We know that gravitational force always acting on the objects. Why these are objects are at rest but not in the free fall? We can deduce that some other forces balance their weights. These forces result from reaction forces of surfaces in contact. The normal (perpendicular to the surface) component of these contact forces is called **normal force** \( F_N \). There is no formula that allows us to calculate normal force. It could be found by solving whole problem. Direction of the normal force is always perpendicular to the surface on which an object is situated. It is directed up only if the surface is horizontal (it is not so, for example, on the incline).

**Tension Force.**

When a cable attached to the object pulls object, we should take into account the **Tension Force** \( F_r \). Direction of this force -- along the cable. There is no
special formula for the tension force. It could be found by solving whole problem. The cables in the problems suppose to be with zero mass and unstretchable. It means that we do not need additional force to accelerate the cable or to stretch it. The cable just transfers force along itself. We will also suppose that when a cable connects objects they are moving with the same magnitude of acceleration. If there are pulleys in a problem, we will suppose that they only change the direction of the force, but not its magnitude. When two objects are attached to each other by cable we should take into account Newton’s 3\textsuperscript{rd} Law of Motion. The forces with which these objects interact through cable are equal in magnitude and directed in opposite directions along the cable.

**Friction Force.**

When object is moving along the solid surface of other object, the atoms of these two surfaces come close enough and interatomic interaction between two surfaces appears. The tangent component of these contact forces is called **Friction Force** \( F_{fr} \). This force could be much smaller than other forces in the problem in question. In this case, you will be informed by conditions of problem that surface is frictionless. In many problems, surface could not be treated as frictionless. The magnitude of the friction force directly proportional to the normal force and depends on the material and quality of a surface that characterized by material constant – the coefficient of friction. If an object is moving, the friction is called **kinetic friction**. In this case, the magnitude of the friction force can be found from the following equation:

\[
F_{fr} = \mu_k F_N \tag{4.8}
\]

Where \( \mu_k \) is the coefficient of kinetic friction. When object is at rest, the friction is called the **static friction**. The magnitude of the friction force in this case can be estimated from the relationship

\[
F_{fr} \leq \mu_s F_N \tag{4.9}
\]

Where \( \mu_s \) is the coefficient of static friction. Usually \( \mu_s > \mu_k \). Both of them are dimensionless because actually they can be treated as a ratio of two forces. The direction of the friction force – always opposite with the respect to direction of motion (in the case of the kinetic friction) or with respect of the possible motion (in the case of the static friction).
4.5. Solving Problems Related to the Newton’s Laws of Motion.

Take a look on the formulas of the Newton’s Laws of Motion (Eq. (4.2) – Eq. (4.6)). We deal with forces. They are vectors directed in space. We need to perform vector addition of vectors directed in different directions of the space. Because of this, it is very important to draw a diagram showing all the forces acting on each object involved (to get right side of the 1st or 2nd Newton’s laws). This diagram is the specific feature of the solving process and it has its own title. It is called

**Free—body Diagram or Force Diagram.** In this diagram, object should be shown free from its environment. This is why the diagram is called free—body diagram.

1. Start consideration of the problem by drawing a sketch of the situation as a whole.

2. Draw a free-body diagram for only one object (at a time). Show all forces acting on this body. Do not show any forces that this body exerts on other objects. Why? Take a look on the left side of the formulas of the 1st and 2nd Laws. Only forces acting on it determine the motion of an object. There is no any mention of the forces that object in question applies to other objects.

3. If several objects are involved, draw a free—body diagram for each object separately.

4. To find a net force, we need to perform a vector addition. We will use the method of components (see the Chapter 3). Therefore we need to introduce coordinate system (again for each object separately). You are absolutely free to choose the direction of a coordinate axes. Usually the most convenient choice is the choice in which one of the axes is directed along of the direction of an object’s motion.

5. Resolve all forces acting on an object into components using formulas (3.2) – (3.3).

6. Try to figure out from the problem’s conditions, what Newton’s Law of Motion should you use. If it is written in the conditions that object is moving with constant velocity or it is at rest, you should apply the 1st Law (use
formulas (4.2a)). If you are given acceleration or you are asked to find it, you should apply 2nd Law (use formulas (4.5)).

7. Write corresponding Newton’s Law equations in components for an object in question. Specify left side of equations (forces acting on the object with their signs – positive if the component is in the positive direction of coordinate axis, otherwise – negative). Any material object experiences the gravity. Formula for the force of gravity is (4.7), direction – always down. If an object situated on the surface, there is normal force, direction – perpendicular to the surface. If there is a cable (string, rope and so on) by which object is pulled, there is tension force directed along the cable. If an object is moving along the surface and there is no mention in conditions that surface is frictionless, if you are given coefficient of friction or you are asked to find it, there is the friction force acting upon an object. Formula for the friction force is given by Eq. (4.8) (if the object at rest you can use for estimation Eq. (4.9)).

8. Writing left side of equations, remember that if there is no motion along considered coordinate axis, the component of acceleration along this axis is zero.

9. Because friction force is determined by the normal force (see Eq. (4.8)). It is useful to diminish the number of unknowns as following. If the X-axis is chosen as a direction of motion and right side of Y-component of equation of motion, we can use this component to express normal forces through other forces and substitute this expression into definition of friction force (Eq. (4.8)) and then put it into X-component of equation of motion. As a result we will diminish the number of equations describing the motion of an object in question.

10. If there are several bodies in the problem, repeat all steps described above for each object separately. If cables connect these objects, apply 3rd Newton’s Law for tension forces acting on the connected objects. Remember, that if objects connected by cables, they are moving with the same magnitude of acceleration.

11. Bring all equations for all objects together and solve this system of equations simultaneously for unknowns.
Actually in this course we will consider usually one or two objects in the system. Some examples of typical problems are given below.

EXAMPLE 4.1. **Apparent weight of an object in an elevator.**
The real weight $W_r$ of an object is the gravitational force that acts on it $W_r = mg$. The apparent weight of the object $W_a$ is the force object exerts on whatever it rests on. Think about apparent weight as the reading on a scale an object is placed on (see Fig. 4-1 on which the object is shown situated on the scale and both of them are inside elevator). It is supposed that scale is situated on the horizontal surface. In this case, $W_a = F_N$. The object’s mass is $m$. Find the apparent weight of this object when: (a) elevator is moving up with constant velocity; (b) elevator is moving up with the acceleration $a$; (c) elevator is moving down with the acceleration $a$; (d) elevator is moving down with the acceleration $a = g$.

There are only two forces acting on the object: gravity force directed down and normal force numerically equal to the apparent weight directed up. They are shown on the Free-Body Diagrams for the cases (a) – (d) in Fig. 4.1. The object is moving along vertical direction. Therefore it is convenient to choose Y-axis of coordinate system directed up. There are no forces directed along X-axis. Because of this we can analyze only Y-component of the Newton’s second law of motion. In each case we need to find normal force $F_N$ that numerically equal to the apparent weight. It could be equal to the real weight (case (a)), greater than the real weight (case (b)), or smaller than real weight (case(c)). In the case (d), apparent weight is zero and object is in the state of weightless. This situation is used to train astronauts. The plane is moving along the special trajectory for which all objects inside this plane experience the state of weightless for some small interval of time.
Fig. 4.1. Example 4.1. Apparent weight of an object in an elevator.

(a) \( \ddot{v} = \text{const}, \ \ddot{a} = 0 \)
\[ \sum F_y = 0 \rightarrow F_N - mg = 0 \]
\[ F_N = mg \]
\[ W_a = W_r \]

(b) \( \ddot{a} \neq 0, \ \ddot{a} > 0 \)
\[ \sum F_y = ma 
\]
\[ F_N = mg + ma \]
\[ W_a < W_r \]

(c) \( \ddot{a} \neq 0, \ \ddot{a} < 0 \)
\[ \sum F_y = -ma 
\]
\[ F_N = mg - ma \]
\[ W_a < W_r \]

(d) \( \ddot{a} = \ddot{g} \)
\[ \sum F_y = -mg \]
\[ F_N = mg - mg = 0 \]
\[ W_a = 0 \]
EXAMPLE 4.2. A 5.0-kg block is at rest on a horizontal surface. The coefficient of friction between the block and the surface is $\mu_k = 0.30$. Constant horizontal force $F$ then acts upon this block and increases its velocity to 10.0 m/s in 5.0 s. (a) Draw the free-body diagram showing all forces acting on the block. (b) What is the acceleration of the block? (c) What is the magnitude of the force? (d) What distance covered by block during 5.0 s of motion?

In this problem, we have again forces acting along vertical axis, that we again choose as Y-axis: normal force and gravity force. But there are now forces acting along horizontal that is convenient to choose as X-axis: external force acting in positive direction of X-axis and frictional force always acting in the direction opposite in the direction of a motion. Free-Body Diagram is shown in the Fig.4.2.

External force acting on the object is constant; therefore the acceleration of the object must be also constant. Acceleration of the object now can be found by using equation for the motion of an object with constant acceleration and information given in conditions.

We will use now both X- and Y-components of the Newton’s second law of motion. It looks as we have two equations and three unknowns: $F$, $F_f$, and $F_N$. But $F_f$ and $F_N$ are related by definition of the frictional force, which we can add to the system of equations. From Y-component of the second law we can find $F_N$ and substitute it into definition of frictional force. Substituting it into X-component of Newton’s second law we will get one equation with one unknown. This allowed us to determine unknown external force $F$. 
\[ m = 5.0 \text{ kg} \]
\[ m_k = 0.30 \]
\[ v_o = 0 \]
\[ v = 8.0 \frac{m}{s} \]
\[ t = 4.0 \text{ s} \]
\[ F = \text{const} \]
\[ x_0 = 0 \]

(a) Free-Body Diagram

(b) \[ v = v_o + at \]
\[ a = \frac{v - v_o}{t} \]
\[ a = \frac{8.0 \frac{m}{s} - 0}{4.0 \text{ s}} = 2.0 \frac{m}{s^2} \]

(c) \[ \sum F_x = ma \rightarrow F - F_f = ma \rightarrow F = m(a + F_f) \]
\[ \sum F_y = 0 \rightarrow F_N - mg = 0 \rightarrow F_N = mg \]
\[ F_f = m_k F_N = m_k mg \]
\[ F = m(a + m_k g) \]
\[ F = (5.0 \text{ kg}) \left[ 2.0 \frac{m}{s^2} + (0.30)(9.80 \frac{m}{s^2}) \right] \]
\[ F = 25 \text{ N} \]

(d) \[ x = x_0 + v_o t + \frac{1}{2} at^2 \]
\[ = 0 + 0 \cdot t + \frac{1}{2}(2.0 \frac{m}{s^2})(4.0 \text{ s})^2 \]
\[ x = 16 \text{ m} \]

Fig. 4-2. Example 4.2.
EXAMPLE 4.3. Consider the mechanical system shown in Figure, where the block $m_1 = 5.00$ kg sliding on a flat frictionless surface is connected by a string over a pulley to the hanging block $m_2 = 8.00$ kg. The system is released from the rest. (a) Draw the free-body diagram for each block. (b) Calculate the acceleration of the system. (c) Find the tension in the string.

There are two objects: object # 1 is situated at the surface; object # 2 is hanging. They are connected through the cable. Besides gravity forces and normal force, the tension force $F_T$ appears in this case. Important new point is that we need to prepare now separately two free-body diagrams. Each object should be considered separately (see Fig. 4.3). It is convenient to choose positive directions of the coordinate axes along directions of motions: X-axis -- to the right for object # 1; Y-axis -- down for the object # 2. There is no motion along Y-axis for the object # 1, and along X-axis for the object # 2. Correspondingly the components of the acceleration vectors along these axes equal zero for object # 1 and object # 2 respectively. There is no influence on motion of the object # 1 from forces acting on it along Y-axis. Therefore there is no necessity to consider Y-component of the Newton’s second law for object # 1. Similar situation is for the X-component of the second law for object # 2. We can reduce number of unknowns taking into account that because of cable connection, two objects are moving with the same acceleration: $a_1 = a_2 = a$, and because of the Newton’s third law of motion $F_{T1} = F_{T2} = F_T$. Equating $F_{T1}$ to $F_{T2}$, we will get one equation with one unknown. Solving it, we will find the acceleration of the system of objects a, and then the tension force $F_T$. 
(a) Free-Body Diagram?
(b) \( a ? \)
(c) \( F_T ? \)

(b) \[ \sum F_x = m_1 a_1 \rightarrow F_{T1} = m_1 a_1 \]
\[ \sum F_y = 0 \rightarrow F_N - m_1 g = 0 \]
\[ a_1 = a_2 = a \]
\[ F_{T1} = F_{T2} = F_T \]
\[ m_1 a = m_2 g - m_2 a \rightarrow a = \frac{m_2}{m_1 + m_2} g \]
\[ a = \left( \frac{8.00 \text{ kg}}{5.00 \text{ kg} + 8.00 \text{ kg}} \right) (9.80 \frac{\text{ m}}{\text{s}^2}) = 6.03 \frac{\text{ m}}{\text{s}^2} \]

(c) \[ F_T = m_1 a = \frac{m_1 m_2}{m_1 + m_2} g \]
\[ F_T = m_2 g - m_2 a = \frac{m_1 m_2}{m_1 + m_2} g \]
\[ F_T = 30.2 \text{ N} \]
EXAMPLE 4.4. Suppose now that, in an Example 4.3, the coefficient of friction between the block \( m_1 \) and the surface is \( \mu_k = 0.400 \). (a) Draw the free-body diagram for each block. (b) Calculate the acceleration of the system. (c) Find the tension in the string.

\[
\begin{align*}
\quad m_1 & = 5.00 \text{ kg} \\
\quad m_2 & = 8.00 \text{ kg} \\
\quad \mu_k & = 0.400 \\
\end{align*}
\]

(a) **Free-Body Diagram?**

(b) \( a \) ?

(c) \( F_T \) ?

\[
\begin{align*}
\sum F_x & = m_1 a_1 \rightarrow F_{T_1} - F_f = m_1 a_1 \\
\sum F_y & = 0 \rightarrow F_N - m_1 g \\
F_N & = m_1 g \\
F_f & = \mu_k F_N = \mu_k m_1 g \\
F_{T_1} - \mu_k m_1 g & = m_1 a_1 \\
F_{T_1} & = m_1 a_1 + \mu_k m_1 g \\
a_1 & = a_2 = a \\
F_{T_1} & = F_{T_2} = F_T \\
\quad m_1 a + \mu_k m_1 g & = m_2 g - m_2 a \\
a & = \frac{m_2 - \mu_k m_1}{m_1 + m_2} g \\
a & = 4.52 \text{ m/s}^2 \\
\end{align*}
\]

(c) \( F_T = m_2 g - m_2 a = m_1 a + \mu_k m_1 g \\
F_T = \frac{(1 + \mu_k) m_1 m_2}{m_1 + m_2} g \\
F_T = 42.2 \text{ N}
\]

Fig. 4.4. Example 4.4.
Besides gravity forces, normal force, and the tension forces, frictional force $F_f$ appears in this example. Frictional force depends on the normal force $F_N$. To decrease numbers of unknown, we can determine normal force from the Y-component of the Newton’s second Law of motion for the object #1 and put it into definition of the frictional force. Then the succession of actions is the same as in the Example 4.3. It is useful to analyze solution by comparing it with the Example 4.3. The comparison shows that introduction of the frictional force decreases the acceleration of the system. It looks reasonable.