1. Prove that $D[x]$ is a domain if and only if $D$ is a domain.

2. Let $p(x) = x^3 + 2x^2 + 2x + 1 \in \mathbb{Z}_7[x]$. Express $p(x)$ as a product of irreducible polynomials in $\mathbb{Z}_7[x]$.

3. Let $R$ be a ring (not necessarily commutative with unity) that contains at least two elements. Suppose that for each nonzero $a \in R$, there exists a unique $b \in R$ such that $aba = a$.
   a) Show that $R$ has no zero divisors.
   b) Show that $bab = b$.

4. a) Let $R$ be a commutative ring with unity. Let $I$ be an ideal of $R$. Suppose that $I$ contains a unit of $R$. Show that $I = R$.
   b) Use part a) to find all ideals of a field $F$.

5. a) Find all the ideals of $\mathbb{Z}_8$.
   b) Find all prime ideals and all maximal ideal of $\mathbb{Z}_8$.

6. Let $R$ be a commutative ring with unity and let $N$ be an ideal of $R$. Let $\sqrt{N} = \{a \in R$ such that $a^n \in N$ for some $n \in \mathbb{Z}^+\}$. Show that $\sqrt{N}$ is an ideal of $R$ (it is called the radical of $N$).

7. Notation is as in the previous exercise.
   a) Show that $N \subset \sqrt{N}$.
   b) Show that if $N$ is a prime ideal, then $N = \sqrt{N}$.
   c) Give an example of a proper ideal $N$ of $R$ such that $N \neq \sqrt{N}$.

8. Let $G$ be a group. Recall that the center $Z$ of $G$ is defined by $Z = \{z \in G \mid zx = xz$ for all $x \in G\}$. We proved that $Z$ is a normal subgroup of $G$. Therefore we can consider the quotient group $G/Z$. Prove that if $G/Z$ is cyclic, then $G$ is Abelian.