1. (§2 Section 34). (7 points)
   Let $\phi: \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{12}$ be the homomorphism such that $\phi(\overline{1}) = \overline{10}$.
   a) Find the kernel $K$ of $\phi$.
   b) Find the group $\phi[\mathbb{Z}_{18}]$.
   c) List the cosets in $\mathbb{Z}_{18}/K$, showing the elements in each coset.
   d) Give explicitly the isomorphism between $\mathbb{Z}_{18}/K$ and $\phi[\mathbb{Z}_{18}]$ given by the First Isomorphism Theorem.

2. (8 points)
   Let $G$ be the group of all real $2 \times 2$ matrices
   $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$, where $ad \neq 0$,
   under matrix multiplication.
   Let $N$ be the subset of $G$ consisting of matrices of the form
   $\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$.
   a) Prove that $N$ is a normal subgroup of $G$ (do not forget to prove that it is a subgroup!).
   b) Prove that $G$ is not Abelian, and that $G/N$ is Abelian.

3. (8 points)
   If $H$ is a subgroup of a group $G$, let $N(H) = \{g \in G \mid gHg^{-1} = H\}$.
   $N(H)$ is called the normalizer of $H$.
   a) Show that $N(H)$ is a subgroup of $G$.
   b) Show that $H \subseteq N(H)$ and that $H$ is normal in $N(H)$.
   c) Show that if $H$ is a normal subgroup of the subgroup $K$ in $G$,
      then $K \subseteq N(H)$ (that is, $N(H)$ is the largest subgroup of $G$ in which $H$ is normal).
   d) Show that $H$ is normal in $G$ if and only if $N(H) = G$.

4. (7 points)
   Let $G$ be a group and let $a \in G$ be an element of finite order $o(a)$.
   a) Prove that if $a^m = e$ for some positive integer $m$, then $o(a)$ divides $m$.
   b) If $N$ is normal in $G$, prove that the order of the coset $aN$ in $G/N$ divides $o(a)$. 

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