Maximum Drawdown Protection

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Abstract

The drawdown of an asset is a risk measure defined in terms of the running maximum $M_t \equiv \max_{s \in [0,t]} S_s$ of the asset’s spot price $S$ over some period $[0, t]$. The asset price $S$ is said to have drawn down by at least $K$ over some period $[0, T]$ if there exists a time $T_D(K) \in [0, T]$, when the stock price was at least $K$ below its maximum-to-date, i.e.: $M_{T_D(K)} - S_{T_D(K)} \geq K$. Similarly, the drawup of an asset is a performance measure defined in terms of the running minimum $m_t \equiv \min_{s \in [0,t]} S_s$ of the asset’s spot price over some period $[0, t]$. The asset price $S$ is said to have drawn up by at least $K$ over some period $[0, T]$ if there exists a time $T_U(K) \in [0, T]$, when the stock price was at least $K$ above its minimum-to-date, i.e.: $S_{T_U(K)} - m_{T_U(K)} \geq K$.

We introduce two digital options whose payoff insures against the adverse movements in market. A digital call written on the maximum drawdown pays $1$ at its maturity date $T < \infty$ if and only if $S$ has drawn down by at least $K$ over the period $[0, T]$. In contrast, a digital option betting on drawing down before drawing up pays $1$ at its maturity date if and only if $S$ draws down by at least $K > 0$ before it draws up by $K$. In this talk, we will begin by constructing model-free static hedges of the latter of the two digital options in terms of one-touch knockouts. We will then proceed to construct semi-static hedges of both options using single-barrier one-touches or vanillas under arithmetic symmetry and continuity assumptions on the underlying process. Finally, we discuss semi-static hedges of the target digital options under geometric symmetry.

This is joint work with Peter Carr and Olympia Hadjiliadis and is a continuation of Peter Carr’s static replication of the digital on maximum drawdown under arithmetic symmetry.