Math2540 / Review for Midterm 3

(1) For each of the following two graphs

\[ G_1 : a \quad b \quad c \quad d \quad e \quad f \]

\[ G_2 : a \quad d \quad b \quad e \quad c \quad f \]

1. List all the circuits.
2. Is there an Euler circuit?
3. Is there an Euler path?
4. Is there a Hamilton circuit?
5. Is there a Hamilton path?
6. Are \( G_1 \) and \( G_2 \) isomorphic?

(2) For the following graphs \( G \),

\[ a \quad b \quad c \quad f \quad d \quad g \quad e \]

(2.a) determine their vertex connectivity \( \kappa(G) \);
(2.b) determine their edge connectivity \( \lambda(G) \);
(2.c) give an Euler circuit.
(3) In the following picture, is it possible to draw a line, without lifting the pencil, which passes through every side exactly once, but does not go through any corner?

Hint: think of as this as the floor plan of a house, where each wall has exactly one door; now make a graph with vertices the rooms and the outside yard, where there is an edge whenever two rooms are connected by a door.

(4) Is it possible to draw the following pictures without lifting one’s pencil (explain!)

(5) Use Dijkstra’s algorithm to find the shortest path from $a$ to $z$ in the following graphs.

and
(6) What is the best solution in the traveling salesperson problem posed by the following graph.

![Graph Diagram](image)

(7) A chain letter asks the recipient to send a copy to five friends. Suppose that 25 people did not comply with this request, and the rest did. If no one got more than one letter, how many people were involved in this scheme?

(8) A computer virus is designed to circumvent anti-virus scans by doing the following: whenever there is a copy in some folder, it looks for three subfolders, puts again a copy of itself in these three folders, and then replaces itself from the current folder by a file named anonymouswashere.txt; whereas if there are less than three subfolders, it doesn’t do anything. When the virus is detected, the complete directory had already been infected, and 43 copies of the file anonymouswashere.txt were found on this computer. How many folders were found containing a copy of the virus?

(9) Give two different isomers of the saturated hydrocarbon \( \text{C}_5\text{H}_{12} \).

(10) Make a binary search tree for the following collection of words (ordered alphabetically)

\[
\text{LAB, CAB, BAN, BED, CUB, BE, AB, A}
\]

(11) Among 75 coins, one is lighter than the others. What is the least number of weighings using a balance scale that are needed to determine the counterfeit? Assuming that we have a full weighing tree (no need to draw it!), determine how many outcomes at the final weighing are impossible.

(12) Make a game tree for the game of Nim with starting position two piles of height 2. Your opponent gives you the choice to go first or second. What should you do?

(13) Draw a binary tree representing the prefix code

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e : 0
a: 10
s: 110
t: 1110
n: 1111
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Then decode the message 1100101111110101.
(14) Create a Huffman code for an alphabet with the following frequencies

\[ A = 0.12 \quad E = 0.23 \quad F = 0.14 \quad O = 0.16 \quad S = 0.35 \]

Then encode the message “SOFA”

(15) Give the prefix and postfix form of the following expressions

(15.a) \[ (\frac{x+2}{5} + 4 \cdot y)^3 + 6 \]

(15.b) \[ ((x + y) + z) - (x + (y + z)) \]

(16) Determine whether the following forms are prefix or postfix and calculate their value.

(16.a) \[ 7 \cdot 2 + 3 \div 5 \cdot 2 \cdot 3 + \ast 2 \uparrow - \]

(16.b) \[ -\div + \uparrow 6 \cdot 2 \uparrow 2 \cdot 5 \cdot 4 \ast 2 \cdot 5 \]

(17) For the following tree, list its vertices in pre-order, in-order and post-order traversal

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(18) For the following tree, draw it as a rooted tree with root \( i \):

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What are the children of \( g \)? What is the arity of this tree? What is its height?

(19) Draw the tree with universal address

\[ 1, 1.1, 1.1.1, 1.1.2, 1.2, 1.3, 1.3.1, 1.3.2, 1.3.3, 2.1, 2.2, 2.3, 2.3.1, 2.3.2, 3, 4, 4.1, 4.1.1 \]