(1) Let $H_1$ and $H_2$ be subgroups of a group $G$. Prove that $H_1 \cap H_2$ is again a subgroup of $G$.

(2) Determine which of the following are group homomorphisms.
   (a) $f : \mathbb{R} \to \mathbb{R} : x \mapsto |x|$
   (b) $g : \mathbb{R}^* \to \mathbb{R}^* : x \mapsto |x|$
   (c) $h : \mathbb{Z} \to \text{SL}(2, \mathbb{R}) : n \mapsto \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$

(3) Construct non-trivial homomorphisms $\mathbb{Z}_3 \to U(18)$ and $\mathbb{Z}_4 \to U(18)$. (Recall that a homomorphism is non-trivial if its image has at least two elements). Can you construct ones with trivial kernel?

(4) What is the order of $5$ in $\mathbb{Z}_{24}$? What is its order in $U(24)$?

(5) Using the method of repeated squares, calculate $121^{293} \mod 500$.

(6) The following is the Cayley graph of $Q_8$, the quaternion group

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<th>$Q_8$</th>
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</table>

Is this group Abelian? What is the order of $j$ in $Q_8$? Show that the permutation $(i \ j \ k)(-i \ -j \ -k)$ (cycle notation), is a group homomorphism.

(7) Write the permutation \( \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 9 & 7 & 8 & 2 & 5 & 4 & 3 \end{pmatrix} \) in cycle form.

Then write it as a product of transpositions and decide whether it is even or odd.

(8) Why are $\mathbb{Z}_4 \oplus \mathbb{Z}_2$ and $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$ not isomorphic?

(9) Let $G$ be an Abelian group and let $T$ be the subset of all elements of finite order in $G$. Prove that $T$ is a subgroup of $G$.

(10) Using the attached flow chart for wallpaper groups, determine the symmetry group of the painting Angels and Devils by Escher: