Part I: Kind of Easy Questions and Problems

1. Which of the following is true:
   
   (a) An American option can be exercised at any time during its life.
   
   (b) Call options will always be exercised at maturity if the asset price is greater than the strike price.
   
   (c) Put options will always be exercised at maturity if the strike price is greater than the asset price.
   
   (d) The value of a short forward at maturity $T$ (when the price of the asset is $S_T$ and the agreed at time zero forward price is $F_{0,T}$) is: $S_T - F_{0,T}$.

2. A trader enters into a one-year short forward contract to sell an asset for $60 when the today's spot price is $58. The spot price in one year proves to be $63. What is the trader's gain or loss? Answer: $3 loss

3. A trader buys 100 European call options with a strike price of $20 and a time to maturity of one year. The cost of each option is $2. The price of the underlying asset proves to be $25 in one year. What is the trader's total gain or loss at maturity? Answer: $300 gain

4. A trader sells 100 European put options with a strike price of $50 and a time to maturity of six months. The price received for each option is $4. The price of the underlying asset is $41 in six months. What is the trader's total gain or loss? Answer: $500 loss

5. The price of a stock is $36 and the price of a three-month call option on the stock with a strike price of $36 is $3.60. Suppose a trader has $3,600 to invest and is trying to choose between buying 1,000 options and 100 shares of stock. How high does the stock price have to rise for an investment in options to be as profitable as an investment in the stock? Answer: $40

6. An interest rate is 15% per annum when expressed with annual compounding. What is the equivalent rate with continuous compounding? Answer: 13.98%

7. An interest rate is 12% per annum when expressed with quarterly compounding. What is the equivalent rate with semiannual compounding? Answer: 12.18%

8. The three-year zero rate is 7% and the four-year zero rate is 7.5% (both continuously compounded). What is the forward rate between the third and the fourth year? Answer: 9%
9. A company invests $1,000 in a five-year zero-coupon bond and $4,000 in a ten-year zero-coupon bond. What is the duration of the portfolio? Answer: 9 years

Hint: The duration of a portfolio $\Pi$ of two bonds $B_1$ and $B_2$: $\Pi = w_1 B_1 + w_2 B_2$ where $w_1$ and $w_2$ are the weights such that $w_1 + w_2 = 1$, is given by:

$$D_\Pi = w_1 D_1 + w_2 D_2$$

where $D_1$ and $D_2$ are the durations of the two bonds. What are the weights in our case?

10. An investor shorts 100 shares when the share price is $50 and closes out the position six months later when the share price is $43. The shares pay a dividend of $3 per share during the six months. What is the investor's profit or loss? Answer: $400 gain

11. The spot price of an investment asset that provides no income is $30 and the risk-free rate for all maturities (with continuous compounding) is 10%.

- What is the three-year forward price? Answer: $40.5
- Answer the previous question assuming that the asset provides an income of $2 at the end of the first year and at the end of the second year. Answer: $35.84

Part II: More Problems ...

12. The price of a European call option that expires in 6 months and has a strike price of $30 is $2. The underlying stock price at time zero is $29 and a dividend of $0.50 is expected in 2 months and again in 5 months. The term structure is flat, that is the risk-free interest rate is 10% and it is the same for all time periods. What is the price of a European put option that expires in six months and has a strike price price of $30?

Hint: Derive a put-call parity relation for stock that pays known dividends. Answer: $2.51

13. A **Bear Spread** or **Put Spread** is an option strategy created using two put options with different strikes $K_1 < K_2$. The bear spread is created by buying a put option with strike price $K_2$ and selling (shorting) a put option with strike $K_1$. Draw the graph of the put spread payoff at maturity as a function of the asset price at maturity $S_T$. If the price today paid for purchasing the put option with strike $K_2$ is $P(K_2)$ and the option price received for selling the put option with strike price $K_1$ is $P(K_1)$, what is the cost today of the put spread.