Financial Derivatives

In this course, we are interested in special tradables called financial derivatives. The financial derivatives derive their values from some underlying tradable(s), which may be a stock, bond, index, interest rate swap, currencies, etc. For the tradable assets underlying financial derivatives, we'll use the notation $S_t$ to represent the price of this underlying as a function of time $t$ and we shall assume that this function is continuous with time, although in reality there are jumps in price between trading days. The payoff of a derivative is thus some function of the underlying asset, $\text{Payoff} = f(S_t)$. Financial derivatives are therefore not independent securities and they may enforce obligations and rights. Examples of financial derivatives are Forwards (linear payoff), Futures and Options (non-linear payoff).

Forwards

Forward Contract is an agreement that enforces an obligation to buy or sell the underlying asset for a certain price specified in the contract, called the forward price, at a certain future date, also specified in the contract called the maturity date. The word obligation here is key. If we enter the forward contract to buy the underlying in the future, we are long forward and if we enter the opposite side of the contract to sell the underlying in the future, then we are short the forward. Entering a forward on either side costs nothing. In contrast, a spot contract is an agreement to buy or sell the asset today. Forward contracts are traded only in the OTC market.

Payoffs from Forward Contracts

If we enter today a long position in a forward, then at the maturity date $T$ (some time in the future) we will be obligated to buy the underlying asset for the forward price $K$ (also called delivery price) specified in the contract, instead of buying the asset at the prevailing market price $S_T$. Keep in mind that, $S_T$, the price of the asset at the future maturity date $T$ is not known today and so, it represents a random variable when viewed from today. Therefore, the payoff $F_T$ from a long position in a forward contract, on one unit of the asset is

$$F_T = f(S_T) = S_T - K$$

payoff from the long position

For example, if $K = 100$ and, when maturity comes, if $S_T = 120$ then if we are long a forward we will be obligated to buy the asset for 100 instead of the higher market price of 120, so our payoff is $F_T = 120 - 100 = 20$ since we buy the asset cheaply. The payoff for the short position of the same forward contract is the negative of our payoff, namely $-(120 - 100) = 100 - 120 = -20$, so in general the payoff for the short position of a forward contract is

$$K - S_T$$

payoff from the short position

The payoffs from long and short positions in a forward contract are clearly linear as functions of the variable $S_T$ and the graphs are mirror images to each other (w.r.t. the $S_T$ axis).
It costs nothing to enter into a forward contract, because you get only obligations, so nobody would want to pay a premium for having obligations as opposed to rights. That's why the payoff at maturity is the total gain or loss for both parties in a forward. It is a zero-sum game in the sense that the payoff of the short side is the negative of the payoff of the long side.

**Options**

Options are traded both on Exchanges and in the OTC market.

**Call Options** give the holder of the option the right but not the obligation to buy the underlying asset at a certain price by a certain date.

**Put Options** give the holder of the option the right but not the obligation to sell the underlying asset at a certain price by a certain date. Option Features:

- the price specified in the contract is called the **strike price** or the **exercise price** and we use the letter $K$ to represent it
- the date specified in the contract is called the **maturity date** or the **exercise date** and we use the letter $T$ to represent it

- **European** options allow you to exercise the option only at maturity
- **American** options allow you to exercise the option at any time up to maturity

Most options traded on exchanges are American. In the exchange-traded equity option market, one option contract is an agreement to buy/sell 100 shares of the underlying asset. The key difference between forwards and options is that in the case of options we have the right but not the obligation to buy/sell the underlying. This means that we would not exercise the option if it is not in our interest to do so. Another difference with forwards is that we have to pay a price to acquire an option. We pay this option premium for the rights that we get. Unlike the forward contract, where both sides are symmetric in that they both
have certain obligations to buy or sell, options have asymmetric sides. We distinguish between a long position and a short position in an option:

- **Long option position** is when we buy an option. It costs money to enter a long option position. We pay a premium for the rights we get by buying the option.

- **Short option position** is when we sell an option. By selling (writing) the option initially, we receive money (the option premium) from the long side of the contract. Unlike the long position, the short position does not have any rights but only the obligation to fulfill the contract in case the long side decides to exercise their rights. The short side gets compensated with the option premium for agreeing to this obligation. Selling an option for the first time is also called writing an option. Keep in mind that the option can be resold afterwards many times at prices determined by the current market values of these options, but only the original underwriter of the option has the obligation to sell (for a call) or buy (for a put) the underlying if the option is exercised by the current holder of the option.

**Long Call Option Payoff**

Compare the definitions of a long forward position and a long call option position. Is the long call option payoff at maturity $= S_T - K$? The answer is no because the long call option comes with rights, so you would choose not to exercise if you would get a negative payoff. This optionality will simply remove the negative payoff from the long forward payoff graph

$$C_T = \begin{cases} S_T - K, & S_T - K > 0 \\ 0, & S_T - K < 0 \end{cases} = \max(0, S_T - K) = (S_T - K)^+$$

The total P&L we obtain by shifting down the payoff diagram above by the price of the option, which the long side pays to the short side.

**Short Call Option Payoff**

The short side of the call option contract has an obligation to sell if the long call side exercises the right to buy. The payoff is simply the negative of the long side payoff:

$$-\max(0, S_T - K) = \min(0, K - S_T)$$
Note that the short call position has either a zero or negative payoff at maturity but it receives the option price for its obligation, so the total P&L (profit/loss) we get by shifting up the payoff diagram above by the price of the option that the short sides receives from the long side.

**Long Put Option Payoff**

Holding a long position in a put option gives us the right to sell the underlying at maturity for the strike price $K$ instead of the spot market price $S_T$, so the payoff is

$$P_T = \begin{cases} 
K - S_T, & K - S_T > 0 \\
0, & K - S_T < 0 
\end{cases} = \max(0, K - S_T) = (K - S_T)^+$$

The total P&L we obtain by shifting down the payoff diagram above by the price of the option, which the long side pays to the short side.

**Short Put Option Payoff**

The short side has a payoff, which is the negative of the long put option payoff:

$$-\max(0, K - S_T) = \min(0, S_T - K)$$

The total P&L we obtain by shifting up the payoff diagram above, by the price of the option, which the
short side receives from the long side.

Homework 1, Due Thursday February 11, 2010

Forwards and Options

1. What is the difference between entering into a long forward with forward price $50 and taking a long position in a call option with strike price of $50?

2. Suppose you sell a put contract on 100 shares with a strike price of $40 and an expiration date in 3 months. The current stock price is $41, which means that the put option is out of the money because if you exercise it right away it would have a negative payoff. The out of the money options are generally cheap. What have you committed yourself to? How much could you gain or lose depending on the final stock price?

3. Suppose you want to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with a strike of $30 costs $2.90. You have $5,800 to invest. Describe two alternative strategies, one involving an investment in the stock and the other involving investment in the option. What are the potential gains and losses from each strategy?

4. Suppose you own 5000 shares worth $25 each. How can put options be used to provide you with insurance against a decline in the value of your stock holding over the next 4 months?

5. Describe with a graph the payoff from the following portfolio: a long forward on an asset and a long Euro put option on the same asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.


7. The current price of a stock is $94 and a 3-month European call options with a strike price of $95 currently sell for $4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2000 call options (= 20 contracts). Both strategies involve an investment of $9,400. What advice would you give? How high does the stock price have to rise for the option strategy to be more profitable?

8. (Job Interview Problem) A trader buys a European call option and sells a European put option at the same time. The options have the same underlying asset, strike price and maturity. Describe the trader's position and visualize the payoff of the portfolio. Under what condition does the price of the call option equal the price of the put?

Hints: Visualize the payoff with diagrams and try to relate this portfolio to a long forward. Keep in mind that the strike price of an option could be anything, in particular it could be equal to the forward price for that maturity. Use the fact that it costs nothing to enter a forward and finally use the Law of One Price.