Introduction

Markets and Financial Derivatives

 Tradable Assets

The key players in finance are the **tradable assets**. Examples of tradables are:

- Stocks: GOOG, AAPL, etc.
- Indices: S&P 500, DJIA, FTSE 100, etc.
- Money Market Instruments: Certificate of Deposit (CD), Eurodollar Deposits, etc.
- Bonds: US Treasury Bills, Notes and Bonds, Corporate Bonds, etc.
- More complex fixed income products: Interest Rate Swaps, etc.
- Currencies: Euro, Yen, British Pound, etc.

Markets

There are two main markets where financial derivatives are traded:

1. **Exchange-Traded Markets**: The Chicago Board of Trade (www.cbot.com), the Chicago Mercantile Exchange (www.cme.com), the Chicago Board Options Exchange (www.cboe.com) etc. At these markets individuals can trade standardized contracts specified by the exchange in a way that counter-party risk is eliminated.

2. **Over-The-Counter Markets** (OTC): Trades are done over the phone or electronically between two financial institutions or between a financial institution and one of its corporate clients, typically a corporate treasurer or fund manager. In this case the terms of the contracts are more flexible but there is always some counter-party risk.

For individuals, there are also online discount brokers where one can open an account and trade stocks, options, futures etc. at discount fees.

Financial Derivatives

In this course, we are interested in special tradables called **financial derivatives**. The financial derivatives derive their values from some underlying tradable(s), which may be a stock, bond, index, interest rate swap, currencies, etc. For the tradable assets underlying financial derivatives, we'll use the notation $S_f$ to represent the price of this tradable as a function of time $t$ and we shall assume that this function is continuous with time, although in reality there are some small jumps in price between trading days. Financial derivatives are therefore not independent securities and they may enforce obligations or rights. Examples of financial derivatives are **Forwards**, **Futures** and **Options**.

Forwards

Forward Contract is an agreement that enforces an **obligation** to buy or sell the underlying asset for a certain price specified in the contract, called the **forward price**, at a certain future date, also specified in the contract called the **maturity date**. The word obligation here is key. If we enter the forward contract to
**buy** the underlying in the future, we are **long forward** and if we enter the opposite side of the contract to **sell** the underlying in the future, then we are **short the forward**. Entering a forward on either side costs nothing. In contrast, a **spot contract** is an agreement to buy or sell the asset today. Forward contracts are traded only in the OTC market.

**Payoffs from Forward Contracts**

If we enter today a long position in a forward, then at the maturity date $T$ (some time in the future) we will be **obligated** to buy the underlying asset for the forward price $K$ (also called **delivery price**) specified in the contract, instead of buying the asset at the prevailing market price $S_T$. Keep in mind that, $S_T$, the price of the asset at the future maturity date $T$ is not known today and so, it represents a **random variable** when viewed from today. Therefore, the payoff $F_T$ from a long position in a forward contract, on one unit of the asset is

$$F_T = S_T - K$$  \hspace{1cm} \text{payoff from the long position}$$

For example, if $K = 100$ and, when maturity comes, if $S_T = 120$ then if we are long a forward we will be obligated to buy the asset for 100 instead of the higher market price of 120, so our payoff is $F_T = 120 - 100 = 20$ since we buy the asset cheaply. The payoff for the short position of the same forward contract is the negative of our payoff, namely $-(120 - 100) = 100 - 120 = -20$, so in general the payoff for the short position of a forward contract is

$$K - S_T$$  \hspace{1cm} \text{payoff from the short position}$$

The payoffs from the long and short positions in a forward contract as a function of the variable $S_T$ are clearly **linear** and the graphs are mirror images to each other (w.r.t. the $S_T$ axis):

![Long and Short Fwd Payoff Graphs](image)

It costs nothing to enter into a forward contract, because you get only obligations, so nobody would want to pay a premium for having obligations as opposed to rights. That's why the payoff at maturity is the total gain or loss for both parties in a forward. It is a **zero-sum game** in the sense that the payoff of the short side is the negative of the payoff of the long side.
Options

Options are traded both on Exchanges and in the OTC market.

Call Options give the holder of the option the right but not the obligation to buy the underlying asset at a certain price by a certain date.

Put Options give the holder of the option the right but not the obligation to sell the underlying asset at a certain price by a certain date. Option Features:

• the price specified in the contract is called the strike price or the exercise price and we use the letter $K$ to represent it

• the date specified in the contract is called the maturity date or the exercise date and we use the letter $T$ to represent it

• European options allow you to exercise the option only at maturity

• American options allow you to exercise the option at any time up to maturity

Most options traded on exchanges are American. In the exchange-traded equity option market, one option contract is an agreement to buy/sell 100 shares of the underlying asset. The key difference between forwards and options is that in the case of options we have the right but not the obligation to buy/sell the underlying. This means that we would not exercise the option if it is not in our interest to do so. Another difference with forwards is that we have to pay a price to acquire an option. We pay this option premium for the rights that we get. Unlike the forward contract, where both sides are symmetric in that they both have certain obligations to buy or sell, options have asymmetric sides. We distinguish between a long position and a short position in an option:

• Long option position is when we buy an option. It costs money to enter a long option position. We pay a premium for the rights we get by buying the option.

• Short option position is when we sell an option. By selling the option we receive money (the option premium) from the long side of the contract. Unlike the long position, the short position does not have any rights but only the obligation to fulfill the contract in case the long side decides to exercise their rights. The short side gets compensated with the option premium for agreeing to this obligation. Selling an option is also called writing an option.

Long Call Option Payoff

Compare the definitions of a long forward position and a long call option position. Is the long call option payoff at maturity $= S_T - K$? The answer is no because the long call option comes with rights, so you would choose not to exercise if you would get a negative payoff. This optionality will simply remove the negative payoff from the long forward payoff graph
More precisely, while we can still buy the underlying at maturity $T$ for the strike price $K$ instead of the spot market price $S_T$, the call option payoff is $C_T = S_T - K$ provided the difference is positive and if it is negative, we will choose not to exercise the option because it would not be in our interest in which case our payoff at maturity is 0. We are ignoring for now the option premium that the long side had to pay. Here is the long call option payoff using three different notations:

$$C_T = \begin{cases} S_T - K, & S_T - K > 0 \\ 0, & S_T - K < 0 \end{cases} = \max(0, S_T - K) = (S_T - K)^+$$

**Short Call Option Payoff**

The short side of the call option contract has an obligation to sell if the long call side exercises the right to buy. The payoff is simply the negative of the long side payoff:

$$-\max(0, S_T - K) = \min(0, K - S_T)$$

Note that the short call position has either a zero or negative payoff at maturity but it receives the option price for its obligation, so the total P&L (profit/loss) we get by shifting up the payoff diagram above by the price of the option.

**Long Put Option Payoff**

Holding a long position in a put option gives us the right to sell the underlying at maturity for the strike price $K$ instead of the spot market price $S_T$, so the payoff is

$$P_T = \begin{cases} K - S_T, & K - S_T > 0 \\ 0, & K - S_T < 0 \end{cases} = \max(0, K - S_T) = (K - S_T)^+$$
**Short Put Option Payoff**

The short side has a payoff, which is the negative of the long put option payoff:

\[
-\max(0, K - S_T) = \min(0, S_T - K)
\]

The total P&L we obtain by shifting up the payoff diagram above by the price of the option, which the short side receives from the long side.

**Principals of Financial Valuation**

Finding the price of a financial asset today is about discounting future cash flows for **time** and for **risk**.

There are two major types of pricing in Finance:

- **Fundamental or Equilibrium Pricing:**
  - price determined by cash flow analysis, supply and demand; for example stock prices, bond prices.

- **Relative or No-Arbitrage Pricing:**
  - price of a target security is determined relative to the known prices of other liquid securities, forming a portfolio replicating the target payoff; for example option prices.

**Law of One Price**

In this course, we shall focus on relative, no-arbitrage pricing in finance, which is based on one fundamental principle, called the **Law of One Price**. This law states that if two portfolios of financial assets have the same future payoffs, date-by-date and state-by-state, then their current prices should be the same. This law usually holds in well-developed markets when liquidity is not an issue. If this law fails, there would be an arbitrage opportunity that would allow for a "free lunch", namely a risk-free profit by
purchasing the cheap portfolio and selling the expensive one at the same time. However, in this case, the market arbitrageurs would take advantage of this opportunity and as a result the law of one price would be restored. Mathematically, the Law of One Price simply says that the pricing functional has to be linear in order to avoid arbitrage opportunities.

Models in Finance

The price of a pizza is determined by the prices of all its ingredients: pizza dough, cheese, tomato sauce, etc. plus the cost of labor and other operational cost. In a similar way, the price of a financial derivative can be determined from the prices of the underlying assets from which its value is derived plus an additional spread reflecting operational cost and a bid-ask spread reflecting supply and demand. One very important feature of the pizza example is the fact that it is model-independent. All we need to price the pizza are the prices of the ingredients and nothing else.

In the case of financial derivatives, the model-independent approach holds when the financial derivatives have linear payoffs in terms of the underlying assets.

As we saw, forward contracts represent such linear derivatives, so no model is needed to price forwards. On the other hand, options are financial derivatives that do not have linear payoffs, so to price an option we do need a model for the underlying asset. For example, it is common to assume the so called Geometric Brownian Motion (GBM) model for the dynamics of the stock underlying an option in order to be able to price this option relative to the price of the stock and a risk-free bond. This is what the famous Black and Scholes formula for pricing call and put options is all about: it prices the option relative to the price of the underlying stock and a risk-free bond, assuming a GBM model for the stock dynamics.

The important thing to realize is that when it comes to derivatives pricing, the models that are used are not meant to predict the future but rather to compare prices of different securities within a common framework. If you are interested in predicting the future then a more appropriate class would be "Crystal Ball Reading for Young Wizards" but it is currently not offered at the College.

Trading Activities

There are three main types of activities traders are engaged with when using financial derivatives:

1. **Hedging** is when a trader has an exposure to the unknown future price of an asset and takes a position in a derivative to offset this exposure and reduce unwanted risk.

2. **Speculation** is when a trader has no exposure to offset but makes a bet on the future price movements based on intuition or fundamental analysis.

3. **Arbitrage** involves taking positions in two or more different markets in an attempt to exploit mispricing and lock in a risk-free profit.

**Homework 1, Due Thursday February 4, 2010**

1. What is the difference between entering into a long forward with forward price $50 and taking a long position in a call option with strike price of $50?

2. Suppose you sell a put contract on 100 shares with a strike price of $40 and an expiration date in 3 months. The current stock price is $41, which means that the put option is out of the money because if you
exercise it right away it would have a negative payoff. The out of the money options are generally cheap. What have you committed yourself to? How much could you gain or lose depending on the final stock price?

3. Suppose you want to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with a strike of $30 costs $2.90. You have $5,800 to invest. Describe two alternative strategies, one involving an investment in the stock and the other involving investment in the option. What are the potential gains and losses from each strategy?

4. Suppose you own 5000 shares worth $25 each. How can put options be used to provide you with insurance against a decline in the value of your stock holding over the next 4 months?

5. Describe with a graph the payoff from the following portfolio: a long forward on an asset and a long Euro put option on the same asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.


7. The current price of a stock is $94 and a 3-month European call options with a strike price of $95 currently sell for $4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2000 call options (= 20 contracts). Both strategies involve an investment of $9,400. What advice would you give? How high does the stock price have to rise for the option strategy to be more profitable?

8. (Challenge Problem) A trader buys a European call option and sells a European put option at the same time. The options have the same underlying asset, strike price and maturity. Describe the trader’s position and visualize the payoff of the portfolio. Under what condition does the price of the call option equal the price of the put?

Hints: Visualize the payoff with diagrams and try to relate this portfolio to a long forward. Keep in mind that the strike price of an option could be anything, in particular it could be equal to the forward price for that maturity. Use the fact that it costs nothing to enter a forward and finally use the Law of One Price.