Solutions for Practice Test 1 for MAT 1375  
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• Problems on Lines. Read section 1.4 from the textbook.

1. Given the equation of the line \( y = x - \frac{(x - 2)}{5} + \frac{3}{5} \)

(a) What is the y-intercept of the line?
(b) What is the slope of the line?

Solution:
(a) The y-intercept we get when \( x = 0 \), so from the equation

\[
y = 0 - \frac{(0 - 2)}{5} + \frac{3}{5} = \frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1
\]

we get that \( y = 1 \) is the y-intercept, as evident from the graph.

(b) The slope is the number in front of \( x \), when we express \( y \) in terms of \( x \):

\[
y = x - \frac{x}{5} - \frac{(x - 2)}{5} + \frac{3}{5} = \frac{5x}{5} - \frac{x}{5} + \frac{2}{5} + \frac{3}{5} = \\
= \frac{5x - x}{5} + \frac{2 + 3}{5} = \frac{4x}{5} + \frac{5}{5} = \frac{4x}{5} + 1
\]

Since the equation of the line is \( y = \frac{4x}{5} + 1 \) we conclude that the slope is \( \frac{4}{5} \).

2. Find the equation of the line passing through the points (1, 3) and (2, 5).

Solution: We need the slope \( m = \frac{(5 - 3)}{(2 - 1)} = \frac{2}{1} = 2 \) and then using the Point-Slope form with the first point, we get the equation of the line: \( y - 3 = 2(x - 1) \)

We can simplify:
• take 3 to the RHS and open the brackets: \( y = 3 + 2(x - 1) = 3 + 2x - 2 = 2x + 1 \)
• final equation \( y = 2x + 1 \)

3. Find the equation of the line passing through the point (2, -1) with slope 3.

Solution: From the Point-Slope form, the equation of the line is:

\[ y - (-1) = 3(x - 2) \]

and then simplify:
• \( y + 1 = 3x - 6 \)
• \( y = 3x - 6 - 1 = 3x - 7 \)
• final equation: \( y = 3x - 7 \)

4. Find the equation of the line that crosses the y-axis at \( y = 1 \) and is perpendicular to the line
2. \(2y - x = 5\).

**Solution:** We know the y-intercept 1, so if the slope is \(m\), then the equation of this line is \(y = mx + 1\). The line \(2y - x = 5\) has a slope of \(\frac{1}{2}\) since if we solve for \(y\) we get:

\[2y = x + 5 \implies y = \frac{1}{2}x + \frac{5}{2}\] The two lines are perpendicular, which means that the product of their slopes is -1: \(m\left(\frac{1}{2}\right) = -1 \implies m = -2\). So, our green line has the equation: \(y = -2x + 1\).

5. Find the equation of the line passing through the point \((-4, 5)\) that is parallel to the line passing through the points \((1, 3)\) and \((-4, 2)\).

**Solution:** Let the line through \((-4, 5)\) has a slope \(m\), then using the Point-Slope form, we get the equation of the line

\[y - 5 = m(x - (-4)) = m(x + 4)\] This line is parallel to the line passing through the points \((1, 3)\) and \((-4, 2)\), so both lines have the same slope \(m\).

We get the slope of the second line from the two points: \(m = \frac{(2 - 3)}{(-4 - 1)} = \frac{(-1)}{-5} = \frac{1}{5}\). So, the equation of our line is

\[y = 5 + \frac{1}{5}(x + 4) = \frac{1}{5}x + 5 + \frac{4}{5} = \frac{1}{5}x + \frac{29}{5}\]

\[y = \frac{1}{5}x + \frac{29}{5}\]

6. Sketch the graph of the line \(3x + y - 1 = 0\)

**Solution:**
Answer questions 7-11 with True or False:

7. The graph of $x = 5y + 6$ has a $y$-intercept of 6. **False!**
8. The graph of $2y - 8 = 3x$ has $y$-intercept 4. **True!**
9. The lines $3x + 4y = 12$ and $4x - 3y = 12$ are perpendicular. **True!**
10. The line in the figure has a positive slope: **False!**

11. The line in the figure has a zero slope: **False!**

- Problems on Circles and Ellipses. Read section 10.1.

12. What is the center and the radius of the given circles:
13. Find the center and the radius of the circle whose equation is given by

- \((x - 2)^2 + (y + 5)^2 = 7\) →
  - center \((2, -5)\)
  - radius \(\sqrt{7}\)

- \(\left(x + \frac{2}{5}\right)^2 + (y + 1)^2 = 16\) →
  - center \(\left(-\frac{2}{5}, -1\right)\)
  - radius \(4\)

- \(\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{5}\right)^2 = \frac{1}{16}\) →
  - center \(\left(\frac{1}{2}, \frac{1}{5}\right)\)
  - radius \(\frac{1}{4}\)

\[x^2 + y^2 + 6x - 4y - 15 = 0\]

- complete square in \(x\)
- \((x + 3)^2 - 24 + y^2 - 4y = 0\)

\[(y - 2)^2 + (x + 3)^2 - 28 = 0\]

- \((y - 2)^2 + (x + 3)^2 = 28\) →

- \(x^2 + y^2 + 25x + 10y = -12\) complete square in \(x\)
- \(\left(x + \frac{25}{2}\right)^2 + y^2 + 10y - \frac{625}{4} = -12\)
Identify the conic section whose equation is given and find its graph. If it is a circle, list its center and radius; if it's an ellipse, list its center, vertices and foci:

14. \[ (x + \frac{25}{2})^2 + y^2 + 10y - \frac{625}{4} = -12 \]  
\[ \text{complete square in } y \]
\[ (y + 5)^2 + \left(x + \frac{25}{2}\right)^2 = \frac{677}{4} \]

\[ (y + 5)^2 + \left(x + \frac{25}{2}\right)^2 = \frac{677}{4} \rightarrow \]

\begin{align*}
\bullet & \quad \frac{(x + 2)^2}{25} + \frac{(y - 1)^2}{4} = 1 \rightarrow \\
\bullet & \quad 16x^2 + 16(y + 2)^2 = 1 \rightarrow \\
\bullet & \quad 3x^2 + 3y^2 = 12 \rightarrow 
\end{align*}
• \( x^2 + y^2 + 6x - 8y + 5 = 0 \) complete square in \( x \)
  \( (x + 3)^2 - 4 + y^2 - 8y = 0 \) complete square in \( y \)

\( (y - 4)^2 + (x + 3)^2 - 20 = 0 \) \( \rightarrow \)

• \( 4x^2 + y^2 + 24x - 4y + 36 = 0 \) complete square in \( x \)
  \( 4(x + 3)^2 + y^2 - 4y = 0 \) complete square in \( y \)

\( (y - 2)^2 + 4(x + 3)^2 - 4 = 0 \) \( \rightarrow \)