1. Add/Subtract the complex numbers and write the result in the standard form $a + bi$, $(a, b$ real):

(a) \[
\left( \frac{2}{3} + \frac{5}{3}i \right) + \left( \frac{1}{3} - \frac{2}{3}i \right) = \frac{2}{3} + \frac{1}{3} + i\left( \frac{5}{3} - \frac{2}{3} \right) = \frac{2 + 1}{3} + i\left( \frac{5 - 2}{3} \right) = 1 + i
\]

(b) \[
\left( \frac{1}{2} - i \right) - \left( \frac{1}{3} - \frac{1}{2}i \right) = \frac{1}{2} - \frac{1}{3} - i + \frac{1}{2}i = \frac{(3 - 2)}{6} - i \left( 1 - \frac{1}{2} \right) = \frac{1}{6} - \frac{1}{2}i
\]

(c) \[
\left( \frac{1}{25} + \frac{2}{5}i \right) - \left( \frac{2}{5} - 2i \right) = \frac{1}{25} - \frac{2}{5} + \frac{2}{5}i + 2i = \frac{(1 - 2\cdot 5)}{25} + i\left( \frac{2}{5} + 2 \right) = \frac{-9}{25} + i\left( \frac{12}{5} \right)
\]

2. Multiply the complex numbers and write the result in the standard form $a + bi$ using that $i^2 = -1$

(a) \[
(2 - 5i) \cdot (3 - 4i) = (2 \cdot 3 - 2 \cdot 4) - (2 \cdot 5 - 20i) = 6 - 8i - 15 + 20i = 6 - 8i + 4i = 6 - 4i
\]

(b) \[
\left( 4 - \frac{3}{2}i \right) \cdot \left( 4 - \frac{1}{2}i \right) = 4 \cdot 4 - \frac{3}{2} \cdot \frac{1}{2}i - \frac{4 \cdot 3}{2}i - \frac{1 \cdot 1}{2}i^2 = 16 - 2i - 6i - \frac{3}{4} = 16 - 8i - \frac{3}{4}
\]

(c) \[
2i\left( \frac{3}{4} - \frac{1}{6}i \right) = \frac{3i}{4} - \frac{2}{6}i^2 = \frac{3i}{4} + \frac{2}{6} = \frac{3i}{4} + \frac{1}{3} + \frac{3}{2}i
\]

(d) \[
(a - b \cdot i)^2 = a^2 - 2ab \cdot i + b^2
\]

3. Divide the complex numbers and write the result in the standard form $a + bi$. Use 2(d)

\[
(a + b \cdot i) \cdot (a - b \cdot i) = a^2 + b^2
\]

(a) \[
\frac{(2 - 7i)}{(3 + 2i)} = \frac{(2 - 7i)\cdot (3 - 2i)}{(3 + 2i)\cdot (3 - 2i)} = \frac{(2 - 7i)\cdot (3 - 2i)}{3^2 + 2^2} = \frac{(2 - 7i)\cdot (3 - 2i)}{13} = \frac{1}{13}(2\cdot 3 - 2\cdot 2i - 3\cdot 7i + 14i^2) = \frac{1}{13}(6 - 4i - 21i - 14) = \frac{1}{13}(-8 - 25i) = -\frac{8}{13} - \frac{25}{13}i
\]

(b) \[
\frac{3i}{2 - 3i} = \frac{3i(2 + 3i)}{(2 - 3i)(2 + 3i)} = \frac{3i(2 + 3i)}{2^2 + 3^2} = \frac{3i(2 + 3i)}{13} = \frac{3i(2i - 3)}{13} = \frac{3}{13}(2i - 3) = \frac{3}{13} - \frac{2}{3}i = -\frac{9}{13} + \frac{6}{13}i
\]

(c) \[
\frac{(2 - 3i)}{3i} = \frac{(2 - 3i)i}{3i^2} = -\frac{1}{3}i(2 - 3i) = -\frac{1}{3}(2i - 3i^2) = -\frac{1}{3}(2i + 3) = -\frac{3}{3} - \frac{2}{3}i = -1 - \frac{2}{3}i
\]

4. Solve the quadratic equations by completing the square:

(a) \[x^2 - 2x - 8 = 0 \rightarrow x^2 - 2x = 8 \rightarrow x^2 - 2x + 1 = 8 + 1 \rightarrow (x - 1)^2 = 9 \rightarrow x = 3, x = -1\]
\[ x - 1 = \pm \sqrt{9} \rightarrow x = 1 \pm 3 \rightarrow x_1 = 1 + 3 = 4, x_2 = 1 - 3 = -2 \]

(b) \( x^2 + 4x + 7 = 0 \rightarrow x^2 + 4x = -7 \rightarrow x^2 + 4x + 2^2 = -7 + 2^2 \rightarrow \)
\[ (x + 2)^2 = -3 \rightarrow x + 2 = \pm \sqrt{-3} \rightarrow x_{1,2} = -2 \pm i\sqrt{3} \]

5. Solve the quadratic equations using the quadratic formula and check your answers using the sum and product formulas for the roots:

Remember the quadratic formula: \( ax^2 + bx + c = 0 \rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

(a) \( n(3n - 10) = 25 \rightarrow 3n^2 - 10n - 25 = 0 \rightarrow n_{1,2} = \frac{-(-10) \pm \sqrt{100 + 300}}{6} = \frac{10 \pm \sqrt{400}}{6} = \frac{10 \pm 20}{6} = n_1 = \frac{10 + 20}{6} = 5, n_2 = \frac{10 - 20}{6} = -\frac{10}{6} = -\frac{5}{3} \)

(b) \( x(x - 2) = -19 \rightarrow x^2 - 2x + 19 = 0 \rightarrow x_{1,2} = \frac{-(-2) \pm \sqrt{4 - 4 \cdot 19}}{2} = \frac{2 \pm \sqrt{4 - 4 \cdot 19}}{2} = \frac{2 \pm \sqrt{-72}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i \)

(c)
\[ n + \frac{3}{n} = \frac{19}{4} \rightarrow n^2 + 3 = \frac{19}{4}n \rightarrow n^2 - \frac{19}{4}n + 3 = 0 \rightarrow n_{1,2} = \frac{-\left(\frac{19}{4}\right) \pm \sqrt{\left(\frac{19}{4}\right)^2 - 4 \cdot 3}}{2} \]
\[ n_{1,2} = \frac{1}{2} \left(\frac{19}{4} \pm \sqrt{\left(\frac{19}{4}\right)^2 - 4 \cdot 3}\right) = \frac{1}{2} \left(\frac{19}{4} \pm \sqrt{\left(\frac{19}{4}\right)^2 - 4 \cdot 3}\right) = \frac{1}{2} \left(\frac{19}{4} \pm \sqrt{\frac{169}{16}}\right) = \frac{1}{2} \left(\frac{19}{4} \pm \frac{13}{4}\right) = \frac{1}{2} \left(\frac{32}{4}\right) = \frac{1}{2} \left(\frac{19}{4}\right) = \frac{1}{2} \left(\frac{19}{4}\right) = \frac{1}{2} (19 \pm 13) \]
\[ n_1 = \frac{(19 + 13)}{8} = 4, n_2 = \frac{(19 - 13)}{8} = \frac{6}{8} = \frac{3}{4} \]

6. Graph the parabolas:

(a) \( y = 2(x - 3)^2 + 1 \rightarrow \)
7. Find the vertex of the parabola and then graph it:

(a) \( y = x^2 + 6x + 8 \) \text{ complete square } \Rightarrow \ y = (x + 3)^2 - 1 \rightarrow

(b) \( y = 2x^2 + 8x + 9 \) \text{ complete square } \Rightarrow \ y = 2(x + 2)^2 + 1 \rightarrow
8. Solve the systems of linear equations by the substitution method as well as the elimination method:

(a) \[
\begin{align*}
3x - 4y &= -25 \\
\end{align*}\]

1. For the elimination method, multiply the first equation by \(-3\) and add the result to the second equation.

Eq. (1) multiplied by \(-3\) is: \[-19y = -19 \rightarrow y = 1\]

then substitute \(y = 1\) into the original 1st equation: \(x + 5 \cdot 1 = -2 \rightarrow x = -7\)

So, the answer is \((-7, 1)\).

2. For the substitution method, we first solve for \(x\) from the first equation: \(x = -2 - 5y\) and substitute this for \(x\) in the 2nd equation:

(b) \[
\begin{align*}
2x + 5y &= 4 \\
5x - 7y &= -29
\end{align*}\]

1. In the elimination method, our goal is to eliminate one of the variables. For example, we can eliminate the \(x\)-variable by multiplying the first equation by 5 and the 2nd equation by \(-2\):

\[
\begin{align*}
10x + 25y &= 20 \\
-10x + 14y &= 58
\end{align*}\]

then we can add the two new equations to get: \(39y = 78 \rightarrow y = 2\)

We next substitute 2 for \(y\) into one of the original equations, say the 1st:

\[2x + 5 \cdot 2 = 4 \rightarrow 2x = -6 \rightarrow x = -3\]

So, the answer is: \((-3, 2)\).

2. In the substitution method, we have to express first one of the variables in terms of the other. Let's solve for \(x\) using the 1st equation:

\[2x = 4 - 5y \rightarrow x = 2 - \frac{5}{2}y\]

then we substitute this for \(x\) in the 2nd equation:

\[5 \left(2 - \frac{5}{2}y\right) - 7y = -29 \rightarrow 10 - \frac{25}{2}y = -29 \rightarrow \frac{(-25 - 7 \cdot 2)}{2}y = -29 - 10 = -39 \rightarrow -\frac{39}{2}y = -39 \rightarrow y = 2\]

and now we can find \(x\) by using one of the original equations, for example, take the first one: \(2x + 5 \cdot 2 = 4 \rightarrow 2x = -6 \rightarrow x = -3\).
9. Bonus type questions:

(a) Find two numbers such that their sum is 10 and their product is 22.

**Solution:** We look for two numbers \( x \) and \( y \) such that \( x + y = 10 \) and \( x \cdot y = 22 \). We can first solve for \( x \):

\[
x = 10 - y
\]

and substitute this into the other equation:

\[
y(10 - y) = 22 \rightarrow 10y - y^2 = 22 \rightarrow y^2 - 10y + 22 = 0
\]

One can then use the quadratic formula to find the solution:

\[
y^2 - 10y + 22 = 0 \quad \text{solutions for } y \quad 5 + \sqrt{3}, 5 - \sqrt{3}
\]

(b) A NYCC student did a job for $360. It took him 6 hours longer than he expected and so he earned $2 per hour less than he expected. How long did he expect that it would take him to do the job?

**Solution:**

Let \( x \) be the time the student expects it would take him to do the job. Then he expects that he would earn a rate of \( \frac{360}{x} \) $ per hour. However, it took him 6 hours longer than he expected, so he actually earned a rate of $2 per hour less than he expected. This is the same as the equation:

\[
\frac{360}{x} - 2 = \frac{360}{x + 6}
\]

To solve this equation, we have to remove the variable from the denominator:

\[
\frac{360 - 2x}{x} = \frac{360}{x + 6} \rightarrow (x + 6)(360 - 2x) = 360x \rightarrow 360x - 2x^2 + 6 \cdot 360 - 12x = 360x \rightarrow 2x^2 + 12x - 6 \cdot 360 = 0 \rightarrow x^2 + 6x - 3 \cdot 360 = 0
\]

and then we use the quadratic formula to find the 2 solutions: \( x^2 + 6x - 3 \cdot 360 = 0 \quad \text{solutions for } x \quad 30, -36

Since the variable \( x \) is time, it has to be positive, so the answer is 30 hours.

(c) In a class of 50 students, the number of female students is 2 more than 5 times the number of male students. How many female students are there in the class?

**Solution:** Let the number of females be \( x \) and the number of males be \( y \), then we have the system of linear equations

\[
\begin{align*}
x + y &= 50 \\
x &= 2 + 5y
\end{align*}
\]

\[
2 + 5y + y = 50 \rightarrow 6y = 48 \rightarrow y = 8 \rightarrow x = 50 - 8 = 42
\]

So, there are 42 female students.