Parity-Protected Josephson Qubits

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Outline

- Qubits: state of the art
- Idea of Parity-Based Protection
- Charge-Pairing Qubits
- Flux-Pairing Qubits
- Superinductors
Physical (faulty) qubit: a two-level quantum system that satisfies DiVincenzo criteria.

Logical (fault-tolerant) qubit: a collection of \( N \) physical qubits that can correct for arbitrary errors in a single qubit by running error correction codes.

The “fault tolerance” theorem: once a \textit{sufficient low error rate} of physical qubits is attained, there is a strategy for correcting errors so that one can carry out indefinitely long computations.
The error rate:

$$\mathcal{E} \equiv \frac{\tau_0}{\tau_d}$$

$$\tau_0$$ - the longest time for one- and two-qubit gates

$$\tau_d \approx \min(T_1, T_\varphi)$$

- the coherence time

Threshold values of $\mathcal{E}$ for different ECCs:

Earlier codes (Steane, Bacon-Shor):

$$\mathcal{E} \approx 10^{-5}$$

Surface correction code (Kitaev et al.):

$$\mathcal{E} \approx 10^{-2}$$
QC “speed” below 1MHz is not practical.
### Qubit Implementations (Isolation ⇔ Speed)

**Trapped \(^{43}\text{Ca}^+\) ions**
- Single-shot readout fidelity 99.93%
- Single-qubit gate fidelity 99.9999%
- Harty *et al.*, *PRL* 113, 220501 (2014)

**Trapped Cs atoms**
- Single-qubit gate fidelity 99.83%
- Xia *et al.*, *PRL* 114, 100503 (2015)

**Quantum dots**
- Single-qubit gate fidelity > 99.5%
- Kim *et al.*, *Nat.QI* 1, 15004 (2015)
- Single-qubit gate fidelity > 99.95%
- Two-qubit gate fidelity 99.5%
- State-of-the-art: Gambetta et al., arXiv:1510.0437

**Superconducting qubits**
- State-of-the-art: Gambetta et al., arXiv:1510.0437
Figure of Merit $= \frac{1}{\varepsilon}$
Superconducting Qubits: State of the Art

Single-qubit gates:
\[ \varepsilon \equiv \frac{\tau_0}{T_2} \sim 10^{-4} \]

Two-qubit gates:
\[ T_2 \approx 40 \ \mu s \]
\[ \tau_0 \approx 40 \ \text{ns} \]

Martinis’ Group, UCSB/Google

\[ \varepsilon = 1 \times 10^{-3} \]
The threshold error rate for the surface error codes is \(~1\%\). If the error rate is 1/10 of \(\varepsilon_{th}\), «a reasonably fault-tolerant logical qubit ... takes \(10^3 - 10^4\) physical qubits».

Physical qubits are macroscopic (\(\geq 100 \mu m\)), areal density \(~10^6\) times less than computer chips - enormous impact on the large scale architecture.

Martinis (UCSB and Google): “We’re somewhere between the invention of the transistor and the invention of the integrated circuit” – not yet...
Simple design of physical qubit (e.g. transmon) does not necessarily eliminate the enormous complexity of logical qubits.

Any design that may help to reduce the complexity of the final product (i.e. the logical qubit) is worth considering.
Classical Dynamics: JC Model and Washboard Potential

\[ \frac{dQ}{dt} + I_C \sin \varphi = I \]

\[ \frac{\hbar C}{2e} \frac{d^2 \varphi}{dt^2} + I_C \sin \varphi = I \]

\[ \omega_p = \sqrt{\frac{2eI_C}{\hbar C}} = \frac{1}{\sqrt{LJC}} \] - Josephson plasma frequency

If time is normalized to \( 1/\omega_J \)

\[ \ddot{\varphi} + \sin \varphi = \frac{I}{I_C} \]

small oscillations at \( \omega_p \)

Motion of a “particle” with coordinate \( \varphi \) and mass \( \propto C \) in a “washboard” potential:

\[ U(\varphi) = E_J \left\{ (1 - \cos \varphi) - \frac{I}{I_C} \varphi \right\} \]
The Josephson junction is a non-dissipative (at $T \to 0$) nonlinear inductor shunted by a capacitor

$\Rightarrow$ a non-linear non-dissipative oscillator.

**Josephson energy**

\[ E_J = \frac{\hbar I_C}{2e} = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_J} \]

**Charging energy**

\[ E_C = \frac{(2e)^2}{2C_J} \]

**Mag. flux quantum**

\[ \Phi_0 = \frac{\hbar}{2e} \approx 20 G \cdot \mu m^2 \]

**Josephson plasma frequency** (typically $\sim 80 GHz \sim 4K$)

\[ \omega_P = \frac{1}{\sqrt{L_J C}} = \frac{\sqrt{2E_J E_C}}{\hbar} \]

- depends on oxide transparency, not on JJ area

**Josephson junction impedance:**

\[ Z_J = \sqrt{\frac{L_J}{C}} \approx 1 \text{k}\Omega \sqrt{\frac{2E_C}{E_J}} \]

- tunable, $E_J/E_C \sim (JJ \text{ area})^2$
Josephson junctions: non-dissipative inductors, their non-linearity allows addressing only the quantum states involved in computation without exciting the rest of the spectrum.

\[ E_J = \frac{\hbar I_C}{2e} = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_J} \]

\[ L = \frac{\hbar}{2eI_C \cos \phi} = \frac{L_J}{\cos \phi} \]

Charging energy

\[ E_C = \frac{(2e)^2}{2C_J} \]

Estimates for Small Junctions

Area: 0.06 \( \mu \)m\(^2\)

Normal-state resistance: 7 k\( \Omega \)

\[ E_J \approx 1K \quad E_C \approx 1.2K \]

Micro energy controls a macro system!
Qubits: state of the art

Idea of Parity-Based Protection

Charge-Pairing Qubits

Flux-Pairing Qubits

Superinductors
Parity Protected Qubits

The goal: to engineer two quantum states *indistinguishable* by the environment.

\[ H = K (X^n + X^{-n}) + V k^2 \]

\( k \) – discrete variable

\( n=2 \) - parity protection

\[ X^\pm 2 |k\rangle = |k \pm 2\rangle \]

\( g \) – the number of discrete components of the lowest-energy even (odd) states

The two-level approximation Hamiltonian

\[ H = \frac{E_{01}}{2} \sigma_z + \frac{E^*}{2} \sigma_x \]

Two almost degenerate low-energy states: \( E_{01} \propto exp(-g) \)
Parity Protected Qubits (cont’d)

- $g$ – the number of discrete components of the lowest-energy even (odd) states

- Decay is suppressed if parity is protected ($E^* = 0$).
- Coupling to “k” noises is suppressed if the envelope decays slowly (large $g$).

- Some fault tolerant rotations can be realized by fast changes of $V(t)$ – protection from the gate pulse noise.

$T_1 = \infty$

long $T_2$
Discrete variable: number of Cooper pairs on the central island

Parity protection (cancellation of “single-particle” tunneling) due to destructive AB interference

The wave function of a charge $q$ moving around a magnetic flux $\Phi_B$ acquires a topological phase shift

$$\Delta \varphi_{AB} = \frac{q}{\hbar} \oint A \cdot dl = \frac{2e}{\hbar} \Phi_B$$

Bell et al. PRL 112, 167001 (2014)
Discrete variable: number of flux quanta in the loop

Correlated tunneling of TWO fluxons

Parity protection (cancellation of “single-particle” tunneling) due to destructive AC interference

Flux(on) Pairing Qubit

The wave function of a particle with magnetic dipole moment $\mu$ moving in 2D around a line charge acquires a topological phase shift proportional to the line charge density.

Bell \textit{et al.} \textit{PRL} \textbf{116}, 107002 (2016)

Aharonov-Casher phase

The toolbox for protected qubits

Novel Josephson elements:

$$\cos(2\varphi)$$

$$\cos(\varphi/2)$$

Superinductors
Qubits: state of the art

Idea of Parity-Based Protection

Charge-Pairing Qubits

Flux-Pairing Qubits

Superinductors
Charge-Pairing Qubits (discrete variable - Cooper pairs number)

Ioffe, Doucot, Feigel’man et. al. (2002 –present)

\[ H = -2E_2 \cos 2\varphi - 2E_1 \cos \varphi + E_C \left( \hat{n} - n_g \right)^2 \]

The two-level approximation Hamiltonian

\[ H = \frac{E_{01}}{2} \sigma_z + \frac{E^*}{2} \sigma_x \]

Perfect symmetry \((E_1 = 0)\):

\[ E_{01} \propto \omega_p \exp(-g) \cos(\pi n_g) \]

\[ \omega_p = 4\sqrt{E_2/E_C} \quad \text{- plasma frequency} \]

\[ g = 4\sqrt{E_2/E_C} \]

Required: \(E_2 \gg E_C\)
Decoupling from noises

Perfect symmetry ($E_1 = 0$):

Energy relaxation is suppressed (very long $T_1$)

Coupling to the charge noise is exp. small
$$\propto \exp(-g)\sin(\pi n_g)$$

Parity violation: single Cooper pair hopping $\delta E_1(t) \neq 0$.

Sources of parity violation: rhombus asymmetry, flux noise.

To suppress coupling to the flux noise, the rhombi chain should be longer.
Fabrication

Multi-angle deposition of Al through a shadow mask.

σ = 2.4%
Microwave Measurements

Multiplexing: several devices with systematically varied parameters.

the device is coupled to the read-out LC resonator

super-L

CPB
Two experimental “knobs”:
- the gate voltage controls \( n_g \) on the central island;
- the flux in the “phase” loop controls \( \varphi \) across the chain.

Global magnetic field:

\[
\begin{align*}
\Phi_R & \approx \Phi_0/2 \\
\varphi &= 2\pi \frac{\Phi}{\Phi_0}
\end{align*}
\]

- phase difference across the chain

Small JJ: \( \sim 0.12 \times 0.12 \ \mu m^2 \)
Large JJ: \( \sim 0.25 \times 0.25 \ \mu m^2 \)
Time-Domain Measurements

Optimal regime: \( \max E_2, \min E_1 \rightarrow \varphi = n\pi \) (min \( E_{01} \))

\[ Q \equiv \omega_{01} T_1 \approx 1 \cdot 10^6 \]

Factor-of-100 suppression of energy relaxation

M. Bell et al. *PRL* 112, 167001 (2014)
Summary

Charge- (flux-) pairing qubits offer the possibility of coherence protection and fault-tolerant operations.

Observed:
- suppression of energy relaxation in a minimalistic rhombi chain;
- spectroscopic evidence of Aharonov-Casher effect in flux-pairing devices.

Current work:
- optimization of the parameters of the flux-pairing qubits (smaller $E_L$ and larger $E_{dps}$);
- development of better superinductors (for applied projects as well as a novel tool to study 1D QPTs).

The toolbox for protected qubits: $\cos(2\varphi), \cos(\varphi/2), \text{super-}L$