1. The probability that an unbalanced coin lands on head is $1/5$. If this coin is tossed twice and each toss is independent answer the following:
   a. List the sample space.
   b. Construct a discrete probability distribution using $X$ as the number of heads of the two-coin toss.
   c. Find $P(X < 1)$, $E(X)$ and $E(1/X)$.

2. a. A random variable $X$ is uniformly distributed between 2 and 12. Find $P[X > 9]$.
   b. A multiple-choice examination has 10 questions each having five possible answers, only one of which is correct. Suppose a student answers all the questions by guessing. What is the probability that she answers at least 9 correctly?

3. $X$ is a normal distribution with standard deviation $\sigma$ and mean $\mu$. Find
   a. $P[x - \mu < 2\sigma]$  
   b. $P[X^2 \leq 1]$ if $\sigma = 8$ and $\mu = 3$.

4. The mean grade on an examination is 72 with standard deviation 8. If the distribution is normal, find the cutoff score of the top 5% the grades.

5. A coin that is claimed to be fair ($p = 1/2$) is flipped 80 times and 45 heads are observed. Can we accept the claim at the 5% significance level?

6. Find the correlation coefficient and the regression line of the points $(-1, -2)$, $(0, 1)$, $(1, 0)$ and $(2, 2)$.

7. An entertainer claims that there are equal numbers of red, white and blue balls in a container. In a random sample of 12 balls, there are 3 red, 5 white and 4 blue balls. Use a chi-square goodness of fit test to check the claim at the 5% significance level.

8. The average amount of tar in a random sample of 25 cigarettes is 12mg with standard deviation of 3mg. The cigarette company claims that the average amount of tar is 11 mg. Can we reject this claim at the 10% significance level?
Solutions.

1. a. $\{hh, ht, th, tt\}$. Note: the events are not equally likely.
   
   b. 
   
   \[
   \begin{array}{c|c}
   x & P(x) \\
   \hline
   0 & 0.64 \\
   1 & 0.32 \\
   2 & 0.04 \\
   \end{array}
   \]

   c. $16/25, 35/25$ and $61/75$

2. a. $f(x) = 0.1$ for $x \in [2, 10]$ and zero elsewhere. $P[X > 9] = 0.3$.
   
   b. $41/5^9$.

3. a. 0.98
   
   b. 0.54

4. 85.1

5. $H_0 : \mu = np = 40$, $H_1 : \mu \neq 40$, $\sigma = \sqrt{np(1-p)} = 2\sqrt{5}$.

   Using the 2- tail test and a normal approximation at the 5% level.

   $z = 1.12 \in (-1.96, 1.96)$, we do not reject the claim.

6. $\hat{y} = -0.3 + 1.1\hat{x}$.

7. $\chi^2 = 0.5$, we do not reject the claim at the 5% level.

8. $H_0 : \mu = 11$ mg, $H_1 : \mu > 11$ mg, $t = 1.67 > 1.32$. Yes we can reject this claim at the 10% level.