Abstract:

Vertex operator algebras are analogues of Lie algebras, in a certain subtle sense. Just as tensor categories of modules for a Lie algebra are fundamentally important, whether or not the Lie algebra is semisimple (in which case every finite-dimensional module is completely reducible), the same is true of tensor categories of modules for a vertex operator algebra. The theory of such tensor categories is deeply related to many themes in mathematics and physics. But while it is a simple matter to define and consider the tensor category of modules for a given Lie algebra, the opposite is true for a given vertex operator algebra, and defining and constructing the appropriate tensor product functors as well as the natural associativity isomorphisms, and proving their necessary properties, are serious problems, requiring suitable hypotheses on the vertex operator algebra and on the module category, and also requiring extensive use of analytic as well as algebraic methods. In recent work with Yi-Zhi Huang and Lin Zhang, we have developed such a theory, called "logarithmic tensor category theory" because the non-semisimplicity of the module category requires logarithms throughout the theory. I will sketch and motivate this work and discuss some applications, and relate it to earlier work of Huang.